King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 371

Final Exam

Term232

23-5

Net Time Allowed: 180 Minutes

MASTER VERSION

1. If the Midpoint method is used to approximate the solution for the following initial-value problem

$$y' = \cos(2t) + \sin(3t),\ 0 \le t \le 1,\ y(0) = 1,$$

with $h = 0.25$, then $w_2 =$ (where $w_i \approx y(t_i)$)

(a) 1.7423 ______ (correct)
(b) 1.25208
(c) -1.53876
(d) 1.53876
(e) 2.01333

2. If the Runge-Kutta method of order four is used to approximate the solution for the following initial-value problem

$$y' = e^{t-y},\ 0 \le t \le 1,\ y(0) = 1,$$

with $h = 0.5$, then $y(0.5) \approx$

(a) 1.2140 ______ (correct)
(b) 0.82449
(c) 0.81212
(d) 0.81111
(e) 0.81333

3. Let $f(0.25) = 1.64872$, $f(0.5) = 2.71828$ and $f(0.75) = 4.48169$. Using Lagrange interpolating polynomials of degrees two, $f(0.43) \approx$

   (a) 2.34886 ______________________________ (correct)
   (b) 2.61667
   (c) 1.82286
   (d) 1.94333
   (e) 2.16667

4. Using Gaussian elimination with backward substitution and two-digit rounding arithmetic to solve the following linear system:

$$\begin{cases} 4x_1 + x_2 + 2x_3 = 9 \\ 2x_1 + 4x_2 - x_3 = -5 \\ x_1 + x_2 - 3x_3 = -9 \end{cases} \text{. (Do not reorder the equations)}$$

   Then $x_3 =$

   (a) 2.9 ______________________________ (correct)
   (b) 3.0
   (c) 3.1
   (d) 3.2
   (e) 3.3

5. Using Gaussian elimination with partial pivoting and three-digit chopping arithmetic to solve $\begin{cases} -6.10x + 5.31y = 47.0 \\ 58.9x + 0.03y = 59.2 \end{cases}$, then $xy =$

(a) 9.98 ____(correct)

(b) 10.0

(c) 9.99

(d) 9.97

(e) 10.1

6. The row interchanges that are required to solve the following linear system $\begin{cases} 5x_1 + x_2 - 6x_3 = 7 \\ 2x_1 + x_2 - x_3 = 8 \\ 6x_1 + 12x_2 + x_3 = 9 \end{cases}$ using partial pivoting are

(a) Interchange rows 1 and 3, then interchange rows 2 and 3 ____________(correct)

(b) Interchange rows 2 and 3 only

(c) Interchange rows 1 and 3 only

(d) Interchange rows 1 and 2, then interchange rows 2 and 3

(e) No need for rows interchange

7. If the coefficient matrix of the linear system

$$2x_1 + x_2 - x_3 = 1$$
$$-4x_1 + 2x_2 + 4x_3 = 0$$
$$6x_1 + 3x_2 + 2x_3 = -5$$

is written in LU form where $L = \begin{bmatrix} 1 & 0 & 0 \\ l_1 & 1 & 0 \\ l_2 & l_3 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$, then $l_2 + u_{22} =$

(a) 7 ______________________________ (correct)
(b) 8
(c) 3
(d) 10
(e) 2

8. Let $x^{(0)} = (0, 0, 0)$. If the Gauss-Seidel method for the system

$$10x_1 - x_2 = 9$$
$$-x_1 + 10x_2 - 2x_3 = 7$$
$$-2x_2 + 10x_3 = 6$$

is performed, then $x_3^{(2)} =$

(a) 0.7899 ______________________________ (correct)
(b) 1.000
(c) 0.740
(d) 0.862
(e) 0.943

9. Let $x^{(0)} = (0, 0, 0, 0)$. If the second iteration of the Jacobi method for the system

$$10x_1 + 5x_2 = 6$$
$$5x_1 + 10x_2 - 4x_3 = 25$$
$$-4x_2 + 8x_3 - x_4 = -11$$
$$-x_3 + 5x_4 = -11$$

is $(x_1^{(2)}, x_2^{(2)}, x_3^{(2)}, x_4^{(2)})$, then $x_3^{(2)} + x_4^{(2)} =$

(a) -2.875 ________________________________ (correct)

(b) -3.001

(c) -2.475

(d) -3.475

(e) -1.475

10. Consider $A = \begin{bmatrix} 1 & 1 & -1 & 2 \\ 2 & 2 & 4 & 5 \\ 1 & -1 & 1 & 7 \\ 2 & 3 & 4 & 6 \end{bmatrix}$. The permutation matrix P such that $P^{-1}A = \begin{bmatrix} 1 & 1 & -1 & 2 \\ 1 & -1 & 1 & 7 \\ 2 & 2 & 4 & 5 \\ 2 & 3 & 4 & 6 \end{bmatrix}$, is

(a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ ________________________________ (correct)

(b) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

11. Given the boundary-value problem

$$y'' = y' + 2y + \cos x, \quad 0 \le x \le \frac{\pi}{2}, \quad y(0) = -0.3, \quad y\left(\frac{\pi}{2}\right) = -0.1.$$

Using the Linear Finite-Difference method with $h = \frac{\pi}{4}$, $y\left(\frac{\pi}{4}\right) \approx$

(a) -0.2829 ____________________________________(correct)

(b) -0.2504

(c) -0.2333

(d) -0.2643

(e) -0.2762

12. Use the Linear Finite-Difference method to approximate the solution of the boundary value problem

$$y'' = x^{-1}y' + 3x^{-2}y + x^{-1}\ln x - 1, \quad 1 \le x \le 2, \quad y(1) = y(2) = 0$$

with $h = 0.2$. If the resulting system of equations is of the form $Aw = b$, then the sum of all elements in the third row of A equals to

(a) $\frac{3}{64}$ ____________________________________(correct)

(b) $\frac{5}{64}$

(c) $\frac{7}{64}$

(d) $\frac{11}{64}$

(e) $\frac{13}{64}$

13. Using Newton's method to find a solution for $\sin x = e^{-x}$ accurate to within 10^{-5} and by taking $p_0 = 3$, then

(a) $p_3 = 3.096364$ ________ (correct)
(b) $p_2 = 0.588533$
(c) $p_3 = 3.0133333$
(d) $p_2 = 3.567841$
(e) $p_2 = 2.867856$

14. Using three-digit rounding arithmetic, $\dfrac{\frac{13}{14} - \frac{6}{7}}{2e - 5.4} =$

(a) 1.80 ________ (correct)
(b) 1.78
(c) 1.95
(d) 2.01
(e) 2.33

15. A natural cubic spline S for a function f is defined on $[1, 3]$ by

$$S(x) = \begin{cases} S_0(x) = a_0 + b_0(x-1) + c_0(x-1)^2 + d_0(x-1)^3 & \text{for } 1 \le x < 2, \\ S_1(x) = a_1 + b_1(x-2) + c_1(x-2)^2 + d_1(x-2)^3 & \text{for } 2 \le x \le 3. \end{cases}$$

If S interpolates the data (1, 1), (2, 1), and (3, 0), then $c_0 + b_1 + d_1 =$

(a) $-\frac{1}{4}$ ________________ (correct)

(b) $\frac{1}{4}$

(c) $-\frac{1}{2}$

(d) $\frac{1}{2}$

(e) $-\frac{3}{4}$

16. Using the Composite Simpson's rule with $n = 4$, $\int_1^2 x \ln x \, dx \approx$

(a) 0.63631 ________________ (correct)

(b) 0.63627

(c) 0.63636

(d) 0.63625

(e) 0.63233

17. Given the following data: $f(0) = 0$ and $f(0.2) = 0.7414$. Using the forward-difference formula and four-digit rounding arithmetic, $f'(0) =$

(a) 3.707 ____(correct)
(b) 3.153
(c) 3.833
(d) 3.531
(e) 3.371

18. Fitting the data: $(0, 1), (0.25, 1.284)\,, (0.5, 1.6487)\,, (0.75, 2.117)\,,$ and $(1, 2.7183)$ with the discrete least squares polynomial of degree at most 2.
Which one of the following equations is one of the resulted normal equations?

(a) $1.875a_0 + 1.5625a_1 + 1.3828a_2 = 4.4015$ ____________________(correct)
(b) $5a_0 + 2.5a_1 + 1.3828a_2 = 8.7680$
(c) $2.5a_0 + 1.875a_1 + 1.5625a_2 = 4.4015$
(d) $5a_0 + 2.5a_1 + 1.5625a_2 = 8.7680$
(e) $5a_0 + 2.5a_1 + 1.5625a_2 = 5.4514$

19. Let $P(x)$ is the least squares polynomial of degree one for the data in the following table

x_i	0	1	2	3	4
y_i	2	3	5	4	6

Then, $P(1) =$

(a) 3.1 ____ (correct)
(b) 3.0
(c) 2.9
(d) 2.8
(e) 2.5

20. If Euler's method is used to approximate the solution for the following initial-value problem

$$y' = e^{t-y},\ 0 \le t \le 1,\ y(0) = 1,$$

with $h = 0.5$, then $y(1) \approx$

(a) 1.43625 ____ (correct)
(b) 1.18393
(c) 1.28093
(d) 1.53333
(e) 1.36667

21. If the divided differences for a function f are given by the following table:

$x_0 = 0$	0		
		$f[x_0, x_1] = \frac{2}{\pi}$	
$x_1 = \frac{\pi}{2}$	$f[x_1]$		$f[x_0, x_1, x_2] = -\frac{4}{\pi^2}$
		$f[x_1, x_2]$	
$x_2 = \pi$	$f[x_2]$		

Then, $f[x_1] + f[x_2] + f[x_1, x_2] =$

(a) $1 - \frac{2}{\pi}$ ________________________________ (correct)

(b) $\frac{2}{\pi} - 1$

(c) $\frac{2\sqrt{2}}{\pi}$

(d) $2 + \frac{2}{\pi}$

(e) $\frac{2}{\pi} - 2$

22. The smallest value of n, required to approximate $\int_0^2 \frac{1}{x+4}\, dx$ by Composite Trapezoidal rule to within 10^{-5}, is

(a) 46 ________________________________ (correct)

(b) 26

(c) 16

(d) 36

(e) 56

23. The relative error in approximations of $p = 10^{\pi}$ by $p^* = 1400$ is

(a) 1.050×10^{-2} ______________________________ (correct)

(b) 1.050×10^{-3}

(c) 1.050×10^{-1}

(d) 1.050×10^{-4}

(e) 1.050×10^{-5}

24. If fixed-point iteration method is used to determine a solution accurate to within 10^{-2} for $x^4 - 3x^2 - 3 = 0$ on [1,2] with $g(x) = \sqrt[4]{3x^2 + 3}$ and $p_0 = 1$, then the approximated solution is

(a) 1.9433 ______________________________ (correct)

(b) 2.0567

(c) 1.8133

(d) 1.1919

(e) 2.3710