King Fahd University of Petroleum and Minerals
Department of Mathematics
Math 371
Exam 1
Term 233
10-7-2024
Net Time Allowed: 90 Minutes

MASTER VERSION

1. If $P_3(x)$ is the third Taylor polynomial for the function $f(x) = 2x\cos(2x) - (x-2)^2$ about $x_0 = 0$, then least upper bound for the error $|f(0.4) - P_3(0.4)|$ is

   (a) 0.05849 (correct)

   (b) 0.08801

   (c) 0.01087

   (d) 0.09090

   (e) 1.11030

2. Suppose p^* must approximate $\sqrt[3]{7}$ with relative error at most 10^{-4}. The largest interval in which p^* must lie is

   (a) $(1.9127398, 1.9131224)$ (correct)

   (b) $(1.0912739, 1.0913122)$

   (c) $(0.9127224, 0.9131398)$

   (d) $(1.9012739, 1.9111224)$

   (e) $(1.9117398, 1.9121224)$

3. Using four-digit rounding arithmetic, $\dfrac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}} =$

   (a) 23.96 _____ (correct)

   (b) 25.33

   (c) 22.27

   (d) 24.82

   (e) 24.01

4. Let $x^3-7x^2+14x-6=0$. Using only the Bisection method on the interval $[3.4, 3.5]$, the approximated solution accurate to within 10^{-2} is

   (a) 3.4187 _____ (correct)

   (b) 3.4140

   (c) 3.4111

   (d) 3.4250

   (e) 3.4176

5. The best (smallest) bound for the number of iterations needed by the Bisection method, to achieve an approximation with accuracy 10^{-3} to solution of $x^3+x-4=0$ lying in $[1,4]$, is

   (a) $n \geq 12$ (correct)

   (b) $n \geq 14$

   (c) $n \geq 10$

   (d) $n \geq 6$

   (e) $n \geq 8$

6. Consider a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^3 - x - 1 = 0$. By taking $p_0 = 1$ and $g(x) = \sqrt{1 + \frac{1}{x}}$, the approximated solution is

   (a) 1.324 (correct)

   (b) 1.235

   (c) 1.257

   (d) 1.706

   (e) 1.057

7. Let $f(x) = 3x^2 - e^x$ on the interval $[0, 1]$. By taking $g(x) = \sqrt{e^x/3}$ and $p_0 = 1$, the minimum number of iteration necessary to obtain an approximation accurate to within 10^{-5} by fixed-point iteration, is

   (a) 16 _____ (correct)

   (b) 27

   (c) 13

   (d) 19

   (e) 11

8. Let $f(x) = -x^3 - \cos x$. If the root of f is approximated by the Secant method with $p_0 = -1$ and $p_1 = 0$, then $p_3 =$

   (a) -1.2521 _____ (correct)

   (b) -0.2529

   (c) -1.6247

   (d) -0.8791

   (e) -0.7666

9. The equation $4x^2 - e^x - e^{-x} = 0$ has two positive solutions. Which of the following initial guess, Newton's method will not work?

   (a) $p_0 = 0$ (correct)
   (b) $p_0 = -3$
   (c) $p_0 = 1$
   (d) $p_0 = 5$
   (e) $p_0 = -5$

10. For $f(x) = \tan(x)$ and letting $x_0 = 0$, $x_1 = 0.6$, and $x_2 = 0.9$. If the second Lagrange polynomial $P_2(x)$ is used to approximate $f(x)$, then $f(0.45) \approx$

   (a) 0.4546 (correct)
   (b) 0.4831
   (c) 0.5333
   (d) 0.4967
   (e) 0.4767

11. Let $P(x)$ be the interpolating Lagrange polynomial for the data $(0, 0), (0.5, m), (1, 3)$ and $(2, 2)$. If the coefficient of x^3 is 6, then $m =$

(a) 4.25 (correct)

(b) 1.75

(c) 3.75

(d) 2.25

(e) 2.75

12. Let $f(8.1) = 16.94410$, $f(8.3) = 17.56492$, and $f(8.6) = 18.50515$. Using the divided difference formula to find the quadratic interpolating polynomial $P_2(x)$, then the coefficient of x^2 in P_2 is

(a) 0.06 (correct)

(b) 0.04

(c) 0.02

(d) 0.08

(e) 0.10

13. Giving the following data:

x_i	-2	-1	0	1
$f[x_i]$	-1	3	m	-1

If the 1st divided difference involving -1 and 0 is given to be $f[-1,0] = -3$, then $m + f[-1,0,1] =$
(where $f[x_0, x_1, x_2]$ is the 2nd divided difference involving x_0, x_1 and x_2)

(a) 1 _____(correct)

(b) 2

(c) 3

(d) 4

(e) 5

14. The relative error in approximation of 9! with $\sqrt{18\pi}(9/e)^9$ is

(a) 9.213×10^{-3} _____(correct)

(b) 1.454×10^{-1}

(c) 1.901×10^{-2}

(d) 7.031×10^{-2}

(e) 3.053×10^{-3}

15. Let $f(x) = \dfrac{e^x - e^{-x}}{x}$. Using three-digit chopping arithmetic, $f(\pi) =$

(a) 7.22 _____(correct)

(b) 7.20

(c) 7.15

(d) 7.17

(e) 7.55