1. Let $f(x) = x\cos(x) - x^2\sin(x)$. Using the most accurate three-point formula and $h = 0.5$, $f'(2.9) + f'(3.1) \approx$

   (a) 13.5468 ________________ (correct)
   (b) 13.5091
   (c) 13.4301
   (d) 13.4029
   (e) 13.3192

2. Given the function $f$ at the following values,

| $x$ | 1.8 | 2.0 | 2.2 | 2.4 | 2.6 |
|---|---|---|---|---|---|
| $f(x)$ | 3.12014 | 4.42569 | 6.04241 | 8.03014 | 10.46675 |

   By using Simpson's rule, $\int_{2.6}^{1.8} f(x)\,dx \approx$

   (a) $-5.03420$ ________________ (correct)
   (b) $-5.02037$
   (c) $-5.08871$
   (d) $-5.01992$
   (e) $-5.01910$

3. If Trapezoidal rule applied to $\int_0^4 f(x)$ gives the value 4 and Simpson's rule gives the value 2, then $f(2) =$

   (a) $\frac{1}{4}$ ______________________________ (correct)

   (b) $\frac{1}{2}$

   (c) $\frac{1}{8}$

   (d) $\frac{1}{6}$

   (e) 1

4. Using the Composite Simpson's rule with $n = 6$, $\int_{-0.5}^{0.5} x \ln(x+1)\, dx \approx$

   (a) 0.088092 ______________________________ (correct)

   (b) 0.088020

   (c) 0.088130

   (d) 0.088421

   (e) 0.088143

5. Let $f(x) = e^{2x}\sin(3x)$ and given that $f''(x)$ has the maximum value on $[0,2]$ at $x = 2$. If the largest value of $h$ required to approximate $\int_0^2 f(x)\,dx$ by Composite Trapezoidal rule to within $10^{-4}$ is $\frac{2}{\alpha}$, then $\alpha =$

   (a) 2168 ______________________________ (correct)
   (b) 2024
   (c) 2215
   (d) 2097
   (e) 2197

6. Which one of the following functions does not satisfy a Lipschitz condition on the given domain $D$?

   (a) $f(t,y) = (1-y^2)^{1/2} + \pi t$ on $D = \{(t,y)|\,0 \le t \le 1 \text{ and } -1 \le y \le 1\}$ ___(correct)
   (b) $f(t,y) = \dfrac{1+y}{1+t}$ on $D = \{(t,y)|\,0 \le t \le 1 \text{ and } -\infty < y < \infty\}$
   (c) $f(t,y) = \dfrac{y}{\pi t} + te^y$ on $D = \{(t,y)|\,1 \le t \le 2 \text{ and } -2 \le y \le 2\}$
   (d) $f(t,y) = -y + ty^{1/2}$ on $D = \{(t,y)|\,2 \le t \le 3 \text{ and } 2 < y < 3\}$
   (e) $f(t,y) = 1 + t\sin(ty)$ on $D = \{(t,y)|\,0 \le t \le 2 \text{ and } -\infty < y < \infty\}$

7. If Euler's method is used to approximate the solution for the following initial-value problem

$$y' = 1 + y/t + (y/t)^2,\ 1 \le t \le 3,\ y(1) = 0,$$

with $h = 0.2$, and knowing that $w(t_8) = 3.0028$ then $y(3) \approx$

(a) 4.5143 ________________________________ (correct)

(b) 5.3333

(c) 4.9806

(d) 5.0333

(e) 5.1056

8. Let $f(x) = 3xe^x - \cos(x)$. If the second derivative midpoint formula is applied with the values of $f(x)$ at $x = 0.8$, 1.3, and 1.8, then $f''(1.3) \approx$

(a) 37.8175 ________________________________ (correct)

(b) 36.5935

(c) 36.6410

(d) 35.9542

(e) 35.4131

9. If $S(x)$ the natural cubic spline that interpolates the data $(-0.25, 1.33203)$ and $(0.25, 0.800781)$, then $S(0.1) =$

   (a) 0.960156 ______________________________ (correct)
   (b) 0.825208
   (c) 0.703876
   (d) 1.238176
   (e) 1.333333

10. If a natural cubic spline $S$ for a function $f$ is defined on $[0, 3]$ by

$$S(x) = \begin{cases} x^3 & \text{for } 0 \le x \le 1, \\ \alpha + \beta(x-1) + \gamma(x-1)^2 - \dfrac{1}{2}(x-1)^3 & \text{for } 1 \le x \le 3, \end{cases}$$

   then $\alpha + \beta + \gamma =$

   (a) 7 ______________________________ (correct)
   (b) 5
   (c) 3
   (d) 1
   (e) 9

11. Suppose $P(x) = 0.125x^2 + a_0$ is the quadratic least square polynomial for the following data $(1,1)$, $(3,\beta)$ . Then, $a_0 =$

(a) 0.875 ________________ (correct)
(b) 0.250
(c) 1.250
(d) 0.725
(e) 0.915

12. Suppose $P(x)$ is the linear least square polynomial for the following data $(-1,-1),(0,1)$, and $(1,\alpha)$. Then, $P(1) =$

(a) $\dfrac{5\alpha+3}{6}$ ________________ (correct)
(b) $\alpha$
(c) $\dfrac{3\alpha+1}{5}$
(d) $\dfrac{3\alpha+2}{4}$
(e) $\dfrac{\alpha+4}{7}$

13. If Euler's method is used to approximate the solution for the following initial-value problem

$$y' = \frac{1+t}{1+y}, \; 1 \le t \le 2, \; y(1) = 2,$$

with $h = 0.5$ and the actual solution to the initial-value is $y(t) = \sqrt{t^2 + 2t + 6} - 1$, then the actual error at $t = 2$ is

(a) 0.033324 ________________________________________________(correct)

(b) 0.023645

(c) 0.063980

(d) 0.078653

(e) 0.008165

14. If you construct the clamped cubic spline $s(x)$ that interpolates the following data: $(1, 1)$ and $(2, 10)$ and knowing that $f'(1) = 2$ and $f'(2) = 20$, then the coefficient of $x^3$ in $s(x)$ is

(a) 4 ________________________________________________(correct)

(b) 3

(c) 6

(d) 8

(e) 5

15. Given the initial-value problem

$$y' = 1 + y/t,\ 1 \leq t \leq 2,\ y(1) = 2,$$

with exact solution $y(t) = t\ln(t) + 2t$. If Euler's method is used to approximate the solution with $h = 0.25$, then the smallest bound for $|y(2) - w_4|$ is (where $w_i \approx y(t_i)$)

(a) $\dfrac{e-1}{8}$ ______ (correct)

(b) $\dfrac{e-1}{10}$

(c) $\dfrac{e^2-1}{8}$

(d) $\dfrac{e^4-1}{8}$

(e) $\dfrac{e^4-1}{4}$