1. If the Midpoint method is used to approximate the solution for the following initial-value problem

$$y' = te^{3t} - 2y,\ 0 \le t \le 1,\ y(0) = 0,$$

with $h = 0.5$, then $y(1) \approx$

(a) 3.1300 ____________ (correct)
(b) 3.3520
(c) 3.5376
(d) 3.3376
(e) 3.1667

2. Consider the following initial-value problem

$$y' = 1 + \frac{t}{y},\ 1 \le t \le 2,\ y(1) = 2.$$

If the Runge-Kutta method of order four is used to approximate the solution for that problem by taking $h = 0.25$, then $w_1 \approx$

(a) 2.3784 ____________ (correct)
(b) 2.0162
(c) 2.5271
(d) 2.8111
(e) 2.6130

3. If linear Lagrange interpolating polynomial through $(0, 1)$ and $(0.75, 2.1170)$ is used to approximate $f(x)$, then $f(0.25) \approx$

   (a) 1.3723 ________________________________(correct)

   (b) 1.6667

   (c) 1.2840

   (d) 1.5433

   (e) 1.2667

4. If Gaussian elimination is used to solve the following linear system

   $4x_1 - x_2 + x_3 = 8$

   $2x_1 + 5x_2 + 2x_3 = 3$,

   $x_1 + 2x_2 + 4x_3 = 11$

   then $x_3 =$

   (a) 3 ________________________________(correct)

   (b) 1

   (c) -1

   (d) 2

   (e) 4

5. Using Gaussian elimination with partial pivoting and one-digit chopping arithmetic to solve $\begin{cases} \pi x + ey = \pi - \dfrac{e}{2} \\ ex + \pi y = e - \dfrac{\pi}{2} \end{cases}$, then $y =$

(a) 0.0 ________________________________ (correct)
(b) 0.5
(c) −0.5
(d) 0.4
(e) −0.4

6. If $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & 0 & a_5 \\ a_6 & 0 & a_7 \end{bmatrix}$ is the coefficient matrix of a linear system that can be written in LU form where $L = \begin{bmatrix} 1 & 0 & 0 \\ l_1 & 1 & 0 \\ l_2 & l_3 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$, then $l_1 + l_3 =$

(a) $\dfrac{a_4^2 + a_1a_6}{a_1a_4}$ ________________________________ (correct)
(b) $\dfrac{a_1^2 + a_4a_6}{a_1a_4}$
(c) $\dfrac{a_1a_4 + a_1a_6}{a_1}$
(d) $\dfrac{a_1a_4 + a_1a_6}{a_1a_2}$
(e) $\dfrac{a_2a_4 + a_1a_6}{a_6a_4}$

7. Let $x^{(0)} = (1,1,1)^t$. If the Gauss-Seidel method for the system

$$4x_1 + 3x_2 = 24,$$
$$3x_1 + 4x_2 - x_3 = 30,$$
$$-x_2 + 4x_3 = -24,$$

is performed, then $x_3^{(1)} =$

(a) -5.0469 ——— (correct)
(b) -5.9667
(c) -4.0332
(d) -3.8621
(e) -4.9438

8. Let $x^{(0)} = (0,0,0,0)^t$. If the second iteration of the Jacobi method for the system

$$4x_1 + x_2 - x_3 + x_4 = -2,$$
$$x_1 + 4x_2 - x_3 - x_4 = -1,$$
$$-x_1 - x_2 + 5x_3 + x_4 = 0,$$
$$x_1 - x_2 + x_3 + 3x_4 = 1,$$

is $(x_1^{(2)}, x_2^{(2)}, x_3^{(2)}, x_4^{(2)})$, then $x_1^{(2)} =$

(a) -0.5208 ——— (correct)
(b) -0.5001
(c) -0.4751
(d) -0.4275
(e) -0.3215

9. Consider $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$. The permutation matrices P_1 and P_2 such that $P_1P_2A = \begin{bmatrix} a_{41} & a_{42} & a_{43} & a_{44} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{11} & a_{12} & a_{13} & a_{14} \end{bmatrix}$, are

(a) $P_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $P_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ ________________ (correct)

(b) $P_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $P_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

(c) $P_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $P_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(d) $P_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $P_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

(e) $P_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $P_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

10. Using the Composite Trapezoidal rule with $n = 4$, $\int_{-0.5}^{0.5} \cos^2 x \, dx \approx$

(a) 0.9119 ________________ (correct)
(b) 0.9302
(c) 0.9363
(d) 0.8725
(e) 0.8889

11. If the Linear Finite-Difference method is used to approximate the solution of the boundary value problem

$$y'' - y = 0, \quad 0 \le x \le 1, \quad y(0) = 1 \quad y(1) = e$$

with $h = \frac{1}{3}$ then $y\left(\frac{2}{3}\right) \approx$

(a) 1.9494 ______________________________ (correct)
(b) 1.9463
(c) 1.4867
(d) 1.9409
(e) 1.7633

12. Using Newton's method to find a solution for $3x = e^x$ and by taking $p_0 = 1.5$, then $p_3 =$

(a) 1.5121 ______________________________ (correct)
(b) 1.5333
(c) 1.5766
(d) 1.5745
(e) 1.5556

13. A natural cubic spline S for a function f is defined on $[1,3]$. Which of the following functions can be that S?

(a) $S(x) = \begin{cases} 2+\frac{3}{4}(x-1)+\frac{1}{4}(x-1)^3 & \text{for } 1 \le x \le 2, \\ 3+\frac{3}{2}(x-2)+\frac{3}{4}(x-2)^2-\frac{1}{4}(x-2)^3 & \text{for } 2 \le x \le 3. \end{cases}$ ______(correct)

(b) $S(x) = \begin{cases} 2+\frac{3}{4}(x-1)-\frac{1}{4}(x-1)^3 & \text{for } 1 \le x \le 2, \\ 3+\frac{3}{2}(x-2)+\frac{3}{4}(x-2)^2-\frac{1}{4}(x-2)^3 & \text{for } 2 \le x \le 3. \end{cases}$

(c) $S(x) = \begin{cases} 2+\frac{3}{4}(x-1)+\frac{1}{4}(x-1)^3 & \text{for } 1 \le x \le 2, \\ 3+\frac{3}{2}(x-2)+\frac{3}{4}(x-2)^2+\frac{1}{4}(x-2)^3 & \text{for } 2 \le x \le 3. \end{cases}$

(d) $S(x) = \begin{cases} 2+\frac{3}{4}(x-1)+\frac{1}{2}(x-1)^3 & \text{for } 1 \le x \le 2, \\ 3+\frac{3}{2}(x-2)+\frac{3}{4}(x-2)^2-\frac{1}{2}(x-2)^3 & \text{for } 2 \le x \le 3. \end{cases}$

(e) $S(x) = \begin{cases} 2+\frac{3}{4}(x-1)-\frac{1}{2}(x-1)^3 & \text{for } 1 \le x \le 2, \\ 3+\frac{3}{2}(x-2)+\frac{3}{4}(x-2)^2-\frac{1}{2}(x-2)^3 & \text{for } 2 \le x \le 3. \end{cases}$

14. Given the boundary-value problem

$$y'' = 2y' - y + x^2 - 1, \quad 0 \le x \le 1, \quad y(0) = 5, \quad y(1) = 10.$$

Using the Linear Finite-Difference method with $h = 0.5$, $y(0.5) \approx$

(a) 7.25 ____(correct)
(b) 5.65
(c) 7.15
(d) 8.75
(e) 8.15

15. Given the following data: $f(1.2) = 11.0231$, $f(1.3) = 13.4637$ and $f(1.4) = 16.4446$. Using the the most accurate three-point formula and four-digit chopping arithmetic, $f'(1.4) \approx$

(a) 32.50 ____ (correct)

(b) 32.15

(c) 31.50

(d) 31.15

(e) 33.37

16. Let $P(x)$ is the least squares polynomial of degree one for the data in the following table

x_i	0	1	2
y_i	2	-3	5

Then, $P(0) =$

(a) -0.1667 ____ (correct)

(b) 1.923

(c) 0.1533

(d) 0.0001

(e) -0.0974

17. Consider this system:
$\begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Then, $x_2 =$

(a) $\dfrac{2l_{11} - l_{21}}{l_{11}l_{22}u_{22}}$ ________________ (correct)

(b) $\dfrac{l_{11} - 2l_{21}}{l_{11}l_{22}u_{22}}$

(c) $\dfrac{l_{11} - l_{21}}{l_{22}u_{22}}$

(d) $\dfrac{l_{11} - 2l_{21}}{l_{22}u_{22}}$

(e) $\dfrac{l_{11} - l_{22}}{l_{21}u_{22}}$

18. If $P_2(x)$ is the second Taylor polynomial of $f(x) = e^x \cos x$ about $x_0 = 0$ is used to approximate $f(0.5)$, then $f(0.5) \approx$

(a) 1.50 ________________ (correct)

(b) 1.25

(c) 1.75

(d) 1.05

(e) 1.00

19. Using Simpson's rule, $\int_0^2 \sin x \, dx \approx$

(a) 1.4251 ________________________________ (correct)
(b) 1.3333
(c) 1.5656
(d) 1.5111
(e) 1.2525

20. Let $f(x) = (x+2)(x+1)x(x-1)^3(x-2)$. To which zero of f does the Bisection method converge when applied on the interval $[-3, 2.5]$?

(a) 2 ________________________________ (correct)
(b) -2
(c) 0
(d) 1
(e) -1