King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 371

Major Exam I: Section 2 and 4

241

October 02 , 2024

Net Time Allowed: 90 Minutes

MASTER VERSION

1. If the second Taylor polynomial $P_2(x)$ approximates $f(x) = x \ln x$ about $x_0 = 1$, then $f(1.5) \approx$

   (a) 0.6250 ____(correct)

   (b) 0.6350

   (c) 0.5250

   (d) 1.5

   (e) 0.5

2. The least upper bound error for approximate $\int_1^{1.5} x \ln x \, dx$ by using the second Taylor polynomial about $x_0 = 1$ is

   (a) 0.0026 ____(correct)

   (b) 0.0020

   (c) 0.0036

   (d) 0.3600

   (e) 0.200

3. Let $f(x) = \dfrac{e^4}{\cos \frac{x}{40} + x}$: using three-digit shopping arithetic, $f(\pi) =$

   (a) 13.000 ________________ (correct)
   (b) 13.1000
   (c) 13.2000
   (d) 13.12
   (e) 13.1

4. Suppose p^* must appropriate 150 with relative error at most 10^{-2}, the largest interval in which p^* must lie is

   (a) (148.5, 151.5) ________________ (correct)
   (b) $(148, 151)$
   (c) (147.5, 151.7)
   (d) (147.4, 151.7)
   (e) (148.6, 151.6)

5. Let $3x = e^x$. Using the bisection method on the interval $[0, 1]$, then $p_3 =$

(a) 0.6250 ____________________ (correct)

(b) 0.6500

(c) 0.6240

(d) 0.6325

(e) 0.6200

6. Let $g(x) = \pi + 0.5 \sin \frac{x}{2}$ on the interval $[0, 2\pi]$ and $p_0 = \pi$. The minimum number of iterations required to achieve 10^{-2} accuracy by fixed point iteration, is

(a) 5 ____________________ (correct)

(b) 4

(c) 6

(d) 7

(e) 3

7. If the secant method used to approximate the solution for the equation $e^x = 3x^2$ with $p_0 = 0$ and $p_1 = 1$, then $p_3 =$

(a) 0.9029 ____(correct)

(b) 0.9929

(c) 0.9129

(d) 0.9329

(e) 0.9429

8. The equation $\sin x = e^{-x}$ has a solution. Using newton's method with $p_0 = 0.5$, the approximation solution to within 10^{-2} then $p_n \approx$

(a) 0.5885 ____(correct)

(b) 0.5850

(c) 0.58985

(d) 0.5785

(e) 0.5403

9. Let $f(x) = \sqrt{x - x^2}$ and $P_2(x)$ be the interpoluation polynomial an $x_0 = 0$, $x_1 = \frac{1}{2}$ and $x_2 = 1$, then $P_2\left(\frac{1}{3}\right) =$

(a) 0.4444 ______________________________ (correct)
(b) 0.4333
(c) 0.4555
(d) 0.5444
(e) 0.3333

10. A damped cubic spline s for a function f is defined on $[1, 3]$ by

$$\begin{cases} s_0(x) = 3(x-1) + 2(x-1)^2 - (x-1)^3, & 1 \le x < 2 \\ s_1(x) = a + b(x-2) + c(x-2)^2 + \frac{1}{3}(x-2)^3, & 2 \le x \le 3 \end{cases}$$

Given $f'(1) = f'(3)$, then $a + b + 4c =$

(a) 4 ______________________________ (correct)
(b) 3
(c) 5
(d) 2
(e) 6

11. Let $f(x) = \ln(x+2) - (x-1)^2$. Using the most accurate three-point formula and $h = 0.2$, $f'(-1.2) + f'(-1.6)$

(a) 13.6236 ____ (correct)

(b) 13.5236

(c) 13.4236

(d) 14.7631

(e) 14.6237

12. Suppose that $f(0) = 1$, $f(0.5) = 2.5$, $f(1) = 2$ and $f(0.25) = f(0.75) = \alpha$. If the composite simpons's rule with $n = 4$ gives the value $\frac{1}{1.2}$ for $\int_0^1 f(x)\,dx$, then $\alpha =$

(a) 0.25 ____ (correct)

(b) 0.5

(c) 0.125

(d) 0.512

(e) 1.25

13. The smallest value of n required to approximate $\int_0^{1.5} \frac{1}{x+4}$ to within 10^{-4}, using composite trapezoidal rule, is

(a) 10 ______________________________ (correct)
(b) 12
(c) 8
(d) 7
(e) 13

14. Given the initial-value problem is

$$y' = \frac{t}{y},\ 0 \le t \le 1,\ 2 \le y \le 3,$$

then the smallest value of Lipschiz constant L is

(a) $\frac{1}{4}$ ______________________________ (correct)
(b) $\frac{1}{3}$
(c) $\frac{1}{2}$
(d) $\frac{1}{5}$
(e) 1