King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 371-03

Major Exam I

241

October 02, 2024

Net Time Allowed: 90 Minutes

MASTER VERSION

1. If $P_2(x)$ is the second Taylor's polynomial for the function $f(x) = e^{-x}\cos x$ about $x_0 = 0$, then the upper bound for the error $|f(0.5) - P_2(0.5)|$ using the error formula is

   (a) 0.04167 ____(correct)

   (b) 0.03228

   (c) 0.05176

   (d) 0.03882

   (e) 0.01006

2. If $P_3(x)$ is the third Taylor's polynomial for the function $f(x) = (x-1)\ln x$ about $x_0 = 1$, then the approximation of $\int_{0.5}^{1.5} f(x)\,dx$ using $\int_{0.5}^{1.5} P_3(x)\,dx$ is

   (a) 0.08333 ____(correct)

   (b) 0.08802

   (c) 0.07802

   (d) 0.06882

   (e) 0.06006

3. Using three-significant digits chopping arithmetic, $-10\,\pi + 6e - \dfrac{3}{62} \approx$

   (a) -15.2 ______ (correct)

   (b) -16.2

   (c) -15.0

   (d) -15.3

   (e) -14.9

4. Given $\sqrt{x} - \cos x = 0$. Perform three iterations (p_1, p_2, p_3) of the Bisection Method on the interval $[0, 1]$, then $p_1 + p_2 + p_3 =$

   (a) 1.875 ______ (correct)

   (b) 0.625

   (c) 0.750

   (d) 1.075

   (e) 0.875

5. Given that $g(x) = \pi + \frac{1}{2}\sin\left(\frac{x}{2}\right)$ has a fixed point in $[0, 2\pi]$. The minimum number of iteration necessary to obtain an approximation which is accurate to within 10^{-2} with $p_0 = 0$ and $K = \frac{1}{4}$ by Fixed-Point Iteration is

(a) $n \geq 4$ ________________ (correct)

(b) $n \geq 3$

(c) $n \geq 2$

(d) $n \geq 7$

(e) $n \geq 6$

6. Let $f(x) = 3x - e^x$. If the root of f is approximated by the Newton's method with $p_0 = 1.5$, then $p_3 \approx$

(a) 1.5121 ________________ (correct)

(b) 0.5121

(c) 1.9121

(d) 0.6121

(e) −1.5121

7. Let $f(x) = e^x$, for $0 \leq x \leq 2$. Using the second Lagrange interpolating polynomial $P_2(x)$ with $x_0 = 0$, $x_1 = 1$ and $x_2 = 2$, then the coefficient of x^2 in $P_2(x)$ is approximately equal to

(a) 1.4763 ________________ (correct)

(b) 2.0119

(c) 2.1170

(d) 1.4003

(e) 1.1119

8. A natural cubic spline $S(x)$ on $[0, 2]$ defined by

$$S(x) = \begin{cases} S_0(x) = 1 + 2x - x^3, & \text{if } 0 \leq x \leq 1, \\ S_1(x) = 2 + b(x-1) + c(x-1)^2 + d(x-1)^3, & \text{if } 1 \leq x \leq 2. \end{cases}$$

Then $b + c + d =$

(a) -3 ________________ (correct)

(b) 0

(c) -1

(d) 1

(e) 3

9. A clamped cubic spline $S(x)$ on $[0,2]$ defined by

$$s(x) = \begin{cases} s_0(x) = 1 + Bx + 2x^2 - 2x^3, & \text{if } 0 \leq x \leq 1, \\ s_1(x) = 1 + b(x-1) - 4(x-1)^2 + 7(x-1)^3, & \text{if } 1 \leq x \leq 2. \end{cases}$$

Using the clamped boundary conditions, the value of $f'(0) + f'(2) =$

(a) 11.0 ____ (correct)

(b) 10.04

(c) 9.02

(d) -10.05

(e) 0.11

10. If the forward-difference formula for derivative of $f(x) = \ln x$ at $x_0 = 1.8$ with $h = 0.01$ is used, then $f'(x_0) =$

(a) 0.55402 ____ (correct)

(b) 0.54798

(c) 0.55540

(d) 0.55556

(e) 0.55006

11. Using Composite Trapezoidal rule with $n = 4$, then $\int_{-2}^{2} x^3 e^x \, dx \approx$

(a) 31.3653 ____ (correct)

(b) 30.6164

(c) 32.2380

(d) 29.2830

(e) 32.2002

12. When approximating $\int_0^{\pi} \cos x \, dx$ by Composite Simpson's rule that will ensure an approximation error within 10^{-4}, then the approximate values of n is

(a) $n \approx 12$ ____ (correct)

(b) $n \geq 9$

(c) $n \geq 8$

(d) $n \geq 5$

(e) $n \geq 20$

13. The relative error in approximation of $p = \sqrt{2}$ with $p^* = 1.414$ is

(a) 1.5101×10^{-4} ______ (correct)

(b) 9.5101×10^{-4}

(c) 1.6011×10^{-2}

(d) 1.3101×10^{-6}

(e) 1.2101×10^{-2}

14. Given $f(t, y) = t^2 y + 1$, if $f(t, y)$ satisfy the Lipschitz condition on $D = \{(t, y)|0 \leq t \leq 1, -\infty \leq y \leq \infty\}$, then the Lipschitz constant L is

(a) 1 ______ (correct)

(b) 0

(c) -1

(d) 2

(e) $\frac{1}{2}$