King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 371

Math371 Exam 1

Term 241

October 02, 2024

Net Time Allowed: 90 Minutes

MASTER VERSION

1. If $P_3(x)$ is the third Taylor polynomial for the function $f(x) = \sqrt{x+1}$ about $x_0 = 0$, then $P_3(0.25)$ is,

   (a) 1.1182 ____ (correct)
   (b) 1.1179
   (c) 1.1192
   (d) 1.1221
   (e) 1.1103

2. Using three-digit rounding arithmetic to calculate, $\dfrac{e^2 - \pi}{\frac{12}{17} + \sin(2)}$,

   (a) 2.62 ____ (correct)
   (b) 2.64
   (c) 2.26
   (d) 3.24
   (e) 2.74

3. Let $f(x) = \sqrt{x} - \cos(x)$. If p_3 is an approximation of the root of $f(x) = 0$ using the Bisection method on the interval $[0, 1]$, then $f(p_3)$ is,

(a) -0.0204 ____ (correct)

(b) 0.0204

(c) 0.0134

(d) -0.0134

(e) -0.170

4. If $g(x) = \frac{1}{2}(x^3 + 1)$ is used, with fixed-point method, to find a root of $x^3 - 2x + 1 = 0$ with $p_0 = 0.5$, then p_3 is equal to,

(a) 0.6022 ____ (correct)

(b) 0.5892

(c) 0.5689

(d) 0.6222

(e) 0.6502

5. The function $g(x) = \pi + \frac{1}{2}\sin(\frac{x}{2})$ has a unique fixed point in the interval $[0, 2\pi]$. Taking $p_0 = \pi$ and using the Corollary 2.5, the minimum number of iteration necessary to obtain an approximation accurate to within 10^{-3} by the fixed-point iteration is,

   (a) 6 ______________________________ (correct)

   (b) 7

   (c) 2

   (d) 10

   (e) 3

6. Let $f(x) = x^2 - 6$. If secant method is used with $p_0 = 3$ and $p_1 = 2$, then p_3 is equal to,

   (a) 2.4545 ______________________________ (correct)

   (b) 2.5454

   (c) 2.4495

   (d) 2.4321

   (e) 2.4500

7. For which initial guess p_0, the Newton's method fails to approximate a root of $\frac{1}{3}x^3 - \frac{5}{2}x^2 + 6x = 0$,

   (a) $p_0 = 3$ ______________________________ (correct)

   (b) $p_0 = 0$

   (c) $p_0 = 1$

   (d) $p_0 = 5$

   (e) $p_0 = 4$

8. Let $x_0 = 0$, $x_1 = 0.5$, and $x_2 = 1$. If the second degree Lagrange polynomial $P_2(x)$ is used to approximate $f(x) = \ln(x+1)$. Then $P_2(0.75)$ is,

   (a) 0.56403 ______________________________ (correct)

   (b) 0.54603

   (c) 0.55962

   (d) 0.55555

   (e) 0.57670

9. If the Natural cubic spline defined on the interval $[0, 2]$ is,

$$S(x) = \begin{cases} 2 + 3x + 2x^3, & \text{if } 0 \le x \le 1, \\ a_1 + b_1(x-1) + c_1(x-1)^2 + d_1(x-1)^3, & \text{if } 1 \le x \le 2, \end{cases}$$

then $a_1 + b_1 + c_1 + d_1$ is equal to,

(a) 20 ______________________________ (correct)
(b) 18
(c) 15
(d) 24
(e) 22

10. Let $f(1) = 1$, $f(1.2) = 1.2625$ and $f(1.4) = 1.6595$. Using most accurate 3-points formulas, the sum $f'(1) + f'(1.2)$ is equal to,

(a) 2.6251 ______________________________ (correct)
(b) 2.6294
(c) 2.6543
(d) 2.2496
(e) 3.4965

11. Let $f(x) = 3xe^{-2x}$. The approximate value of $f''(0.5)$ with $h = 0.1$ is equal to,

(a) -2.2294 ______________________________ (correct)
(b) -0.2229
(c) -2.5423
(d) 2.2596
(e) -2.9654

12. Approximate value of the integral $\int_0^2 e^{-2x}\cos(3x)\,dx$ using the composite Trapezoidal rule with $n = 4$ is,

(a) 0.19517 ______________________________ (correct)
(b) 0.21324
(c) 0.29757
(d) 0.15571
(e) 0.20547

13. The smallest value of n required to approximate the integral $\int_1^3 x^2 \ln(x)\, dx$ by composite Simpson's rule within 10^{-4} is,

(a) 8 ______ (correct)
(b) 6
(c) 10
(d) 12
(e) 9

14. Which of the following functions satisfy the Lipschitz condition on the given domain.

(a) $f(t,y) = ty + \sin(ty)$ on $D = \{(t,y)|\ 0 \le t \le 4, 0 \le y \le 4\}$ ______ (correct)
(b) $f(t,y) = t^2 \ln(y)$ on $D = \{(t,y)|\ 1 \le t \le 2, 0 \le y \le 2\}$
(c) $f(t,y) = \frac{\sin(t)}{(1+y)}$ on $D = \{(t,y)|\ 0 \le t \le \pi, -1 \le y \le 1\}$
(d) $f(t,y) = 2t + \tan(y)$ on $D = \{(t,y)|\ 1 \le t \le 2, 0 \le y \le \pi\}$
(e) $f(t,y) = 2t^2 + \sqrt{1-y^2}$ on $D = \{(t,y)|\ 0 \le t \le 2, -2 \le y \le 2\}$