King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 371

Major Exam II

241

November 06 , 2024

Net Time Allowed: 90 Minutes

MASTER VERSION

1. If Euler's method is used to approximate the solution for the following initial-value problem

$$y' = \frac{1+t}{1+y}, \quad 1 \le t \le 2, \quad y(1) = 2,$$

with $h = 0.25$, then $y(1.5) \approx$

(a) 2.3443
(b) 2.2199
(c) 2.3941
(d) 2.2978
(e) 2.4101

2. Given the initial-value problem

$$y' = \frac{2}{t}y + t^2 e^t, \quad 1 \le t \le 2, \quad y(1) = 0,$$

with exact solution $y(t) = t^2(e^t - e)$. If Euler's method is used to approximate the solution with $h = 0.1$, then the smallest bound for $|y(2) - w_{10}|$ is

(a) 15.65
(b) 1.565
(c) 0.1565
(d) 0.0156
(e) 156.54

3. If the Runge-Kutta method of order four is used to approximate the solution for the following initial-value problem

$$y' = e^{t-y}, \quad 0 \leq t \leq 1, \quad y(0) = 1,$$

with $h = 0.2$, then $y(0.2) \approx$

(a) 1.0783
(b) 1.2135
(c) 0.9378
(d) 0.9851
(e) 0.8510

4. Using Gaussian elimination with partial pivoting and three-digit chopping arithmetic to solve

$$\begin{cases} -6.10x + 5.31y = 47.0, \\ 58.9x + 0.03y = 59.2, \end{cases} \quad \text{then } xy =$$

(a) 9.98
(b) 10.1
(c) 9.89
(d) 10.2
(e) 8.91

5. If Gaussian elimination with backward substitution (without pivoting) is used to solve the linear system $\begin{cases} 2x_1 + 3x_2 - x_3 = 2, \\ 4x_1 + 4x_2 - x_3 = -1, \\ -2x_1 - 3x_2 + 4x_3 = 1, \end{cases}$ then $x_1 - x_2 + x_3 =$

(a) -5
(b) -7
(c) -6
(d) -8
(e) 6

6. If the matrix $\begin{bmatrix} 2 & 1 & -1 \\ -4 & 2 & 4 \\ 6 & 3 & 2 \end{bmatrix}$ is expressed in LU form, where $L = \begin{bmatrix} 1 & 0 & 0 \\ l_1 & 1 & 0 \\ l_2 & l_3 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$, then $l_1 + u_{33} =$

(a) 3
(b) 5
(c) 7
(d) -4
(e) 4

7. If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$ and $PA = \begin{bmatrix} a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \\ a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix}$, then the permutation matrix $P =$

   (a) $P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

   (b) $P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

   (c) $P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

   (d) $P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & & 1 & 0 \end{bmatrix}$

   (e) $P = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

8. If $\mathbf{x} = (1.827586, 0.6551724, 1.965517)^t$ and $\bar{\mathbf{x}} = (1.8, 0.64, 1.9)^t$ are the actual and approximate solutions for a certain linear system, respectively. Then the distance between the actual and approximate solutions with respect to the l_∞ norm is

   (a) 0.065517
   (b) 0.0151724
   (c) 0.027586
   (d) 0.034205
   (e) 0.047851

9. The sum of all eigenvalues of the matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ -4 & 6 & 2 \\ 16 & -15 & -5 \end{bmatrix}$ is

(a) 4
(b) 3
(c) 2
(d) 5
(e) 6

10. Let $\mathbf{x}^{(0)} = (1, 1, 1)^t$. If the Gauss-Seidel method for the system $\begin{cases} 4x_1 + 3x_2 = 24, \\ 3x_1 + 4x_2 - x_3 = 30, \\ -x_2 + 4x_3 = -24, \end{cases}$ is performed, then $x_1^{(1)} + x_2^{(1)} =$

(a) 9.0625
(b) 8.1296
(c) 7.0321
(d) 11.7215
(e) 6.4428

11. Let $\mathbf{x}^{(0)} = (0,0,0,0)^t$. If the second iteration of the Jacobi method for the system $\begin{cases} 4x_1 + x_2 - x_3 + x_4 = -2, \\ x_1 + 4x_2 - x_3 - x_4 = -1, \\ -x_1 - x_2 + 5x_3 + x_4 = 0, \\ x_1 - x_2 + x_3 + 3x_4 = 1, \end{cases}$ is $(x_1^{(2)}, x_2^{(2)}, x_3^{(2)}, x_4^{(2)})^t$, then $x_2^{(2)} =$

(a) -0.0416
(b) -0.0475
(c) -0.0506
(d) -0.0517
(e) -0.0826

12. The linear least square polynomial for the data below is:

x_i	0	1	2	3	4
y_j	1.1	1.6	2.3	2.7	3.1

(a) $1.14 + 0.51x$
(b) $1.05 + 0.61x$
(c) $1.11 + 0.59x$
(d) $1.05 + 0.63x$
(e) $1.07 + 0.59x$

13. Suppose $P(x)$ is the linear least square polynomial for the following data $(0,1)$,$(1,4)$, and $(2,6)$. Then, the coefficient of x in $P(x)$ equals to

(a) 2.5
(b) 3.1
(c) 5.2
(d) 3.7
(e) 1.9

14. Let $\mathbf{x}^{(0)} = (0,0)^t$. If the first iteration of the conjugate gradient method for the system $\begin{cases} 0.1x_1 + 0.2x_2 = 0.3, \\ 0.2x_1 + 113x_2 = 113.2, \end{cases}$ is $(x_1^{(1)}, x_2^{(1)})$, then $x_1^{(1)} + x_2^{(1)} =$

(a) 1.0044
(b) 1.0318
(c) 1.0059
(d) 1.0725
(e) 1.1058