King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 371

Major Exam II, Sections: 2 and 4

241

November 06, 2024

Net Time Allowed: 90 Minutes

MASTER VERSION

1. If Euler's method is used to approximate the solution of the initial-value problem $y' = -y + ty^{\frac{1}{2}}$, $2 \le t \le 3$, $g(2) = 2$, with $h = 0.5$, then $y(3) \approx$

   (a) 3.1493 ______________________________ (correct)
   (b) 3.4593
   (c) 3.8493
   (d) 2.8321
   (e) 2.1443

2. Given the initial-value problem

$$y' = 1 + \frac{y}{t},\ 1 \le t \le 2,\ y(1) = 2,\ \text{ with } h = 0.1,$$

   with exact solution $y(t) = t \ln t + 2t$. If the Euler's method is used to approximate the solution, then the least bound for $|y(2) - w_{10}|$ is
   $(w_i \approx y(t_i))$

   (a) 0.0859 ______________________________ (correct)
   (b) 0.0239
   (c) 0.0135
   (d) 0.1585
   (e) 0.0231

3. If the Midpoint method is used to approximate the solution for the initial-value problem $y' = y - t^2 + 1$, $0 \le t \le 2$, $y(0) = 0.5$, $h = 0.2$, then $w_2 =$

   (a) 1.2114 ____(correct)
   (b) 1.8104
   (c) 1.2351
   (d) 1.5114
   (e) 1.2714

4. If the Runga-Kutta method of order four is used to approximate the solution for the initial-value problem $y' = 1 + \dfrac{y}{t}$, $1 \le t \le 2$, $y(1) = 2$, with $h = 0.25$, then $w_1 =$

   (a) 2.7789 ____(correct)
   (b) 2.1389
   (c) 2.5329
   (d) 0...
   (e) 3.4789

5. Given the linear system

$$\begin{aligned} x - y + \alpha z &= -2 \\ -x + 2y - \alpha z &= 3 \\ \alpha x + y + z &= 2 \end{aligned}$$

then the value for α for which the system has no solution is

(a) 1 ____ (correct)

(b) -1

(c) 0

(d) 2

(e) -2

6. The linear system $\begin{bmatrix} x - y = 2 \\ 2x + 2y + 3z = -1 \\ -x + 3y + 2z = 4 \end{bmatrix}$ in the matrix form $Ax = b$ and $A = Lu$ form then the solution for $Lw = b$ is

(a) $\left(2, -5, \frac{17}{2}\right)^t$ ____ (correct)

(b) $\left(2, 0, \frac{17}{2}\right)^t$

(c) $(1, 0, 1)^t$

(d) $\left(3, -5, \frac{17}{2}\right)^t$

(e) $(1, 0, 2)^t$

7. If P is the permutation matrix so that PA can by factored into the product Lu, where L is lower triangular with $1s$ on its diagonal and u is upper triangular for the matrix $A = \begin{bmatrix} 0 & 2 & -1 \\ 1 & -1 & 2 \\ 1 & -1 & 4 \end{bmatrix}$, then $P_* \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} =$

(a) $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ ______ (correct)

(b) $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$

(d) $\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$

(e) $\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$

8. The linear system

$$\begin{aligned} x + 2y &= 3 \\ 1.001x - y &= 0.001 \end{aligned}$$

has $X = (1, 1,)^t$ as the actual solution and
$\hat{X} = (1.01, 0.98)^t$ as an approximate solution then $\| A\hat{X} - b \|_\infty =$

(a) 0.03 ______ (correct)

(b) 1.01

(c) 0.33

(d) 0.31

(e) 0.01

9. If $A = \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix}$,then l_2 norm of A is equal to

   (a) 3.1623 ____(correct)
   (b) 0
   (c) 10
   (d) 4.1823
   (e) 3.5613

10. The second iteration of the Jacobi Method for the system

$$\begin{aligned} -2x_1 + x_2 + \tfrac{1}{2}x_3 &= 4 \\ x_1 - 2x_2 - \tfrac{1}{2}x_3 &= -4 \\ x_2 + 2x_3 &= 0 \end{aligned}$$

   is $(x_1^{(2)}, x_2^{(2)}, x_3^{(2)})$ with $X^0 = (0,0,0)^t$, then $x_1^{(2)} + x_2^{(2)} =$

   (a) 0 ____(correct)
   (b) 1
   (c) 2
   (d) 3
   (e) 4

11. The second iteration of the Gauss-Seidel method for the system

$$
\begin{aligned}
-2x_1 + x_2 + \tfrac{1}{2}x_3 &= 4 \\
x_1 - 2x_2 - \tfrac{1}{2}x_3 &= -4 \\
x_2 + 2x_3 &= 0
\end{aligned}
$$

is $(x_1^{(2)}, x_2^{(2)}, x_3^{(2)})$ with $x^{(0)} = (0,0,0)^t$, then $x_1^{(2)} =$

(a) -1.625 ________________ (correct)

(b) 0

(c) -2.635

(d) -1.325

(e) -1

12. The linear system

$$
\begin{aligned}
x_1 + 2x_2 &= 3 \\
x_1 - x_2 &= 0
\end{aligned}
$$

has solution $(x_1, x_2)^t = (1,1)^t$ the two steps of the conjugate gradient with $C = C^{-1} = I$ of the system (use two-digit choppling, $x^{(0)} = 0$

(a) $(2,1)^t$ ________________ (correct)

(b) $(1,1)^t$

(c) $(1,1.5)^t$

(d) $(2,1.5)^t$

(e) $(2.5,1)^t$

13. Let $P(x)$ is the least squares polynomial of degree one for the following data

$$(3, 8.3),\ (5, 11.3),\ (8, 14.4),\ \text{and}\ (10, 15.9),$$

then $p(5) =$

(a) 10.8586 ________________________________ (correct)
(b) 11.3
(c) 11.8357
(d) 10.4587
(e) 11.1387

14. Suppose that

$$\begin{aligned} -6x + \alpha y + z &= 2 \\ 3x + y - z &= 0 \\ 3x\left(-\tfrac{1}{2}\alpha + 2\right) y + z &= 1 \end{aligned}$$

For which of the following values of α will be row change required when solving this system using partial priority.

$$\mathrm{I} : \alpha = -8,\ \ \mathrm{II} : \alpha = -5,\ \ \mathrm{III} : \alpha = 1$$

(a) II and III ________________________________ (correct)
(b) II only
(c) III only
(d) I and II
(e) I and III