1. Given the nonlinear system

$$3x_1^2 - x_2^2 = 0$$
$$x_1 x_2^2 - x_1^3 = 1$$

Using the method of steepest Descent with $X^{(0)} = [1,1]^t$ and $\alpha = 0.01$, then $X^{(1)} \approx$

(a) $[0.72, 1.12]^t$
(b) $[0.6, 1.16]^t$
(c) $[0.8, 1.06]^t$
(d) $[0.92, 1.06]^t$
(e) $[0.82, 1.16]^t$

2. Given the nonlinear system

$$3x_1^2 - x_2^2 = 0$$
$$x_1 x_2^2 - x_1^3 = 1$$

Using the Newton's method with $X^{(0)} = [1,1]^t$ then $X^{(1)} \approx$

(a) $[0.75, 1.25]^t$
(b) $[0.72, 1.06]^t$
(c) $[0.85, 1.75]^t$
(d) $[0.75, 1.06]^t$
(e) $[0.95, 1.75]^t$

3. Given the following data $(-1,0)$, $(0,1)$, $(1,4)$. Using Singular value decomposition to determine the least squares polynomial $P_1(x) = a_1x + a_0$, then $P_1\left(\frac{1}{2}\right) \approx$

   (a) $\frac{8}{3}$
   (b) $\frac{7}{3}$
   (c) 2
   (d) $\frac{5}{3}$
   (e) 1

4. Which of the following matrices are orthogonal matrix

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & -1 \\ \frac{1}{\sqrt{2}} & 1 \end{bmatrix}, \; B = \begin{bmatrix} 1 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix}, \; C = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

   (a) C only
   (b) A only
   (c) B only
   (d) A and C
   (e) B and C

5. Given the linearly independent vectors

$$X^{(1)} = (2,0,0)^t,\ X^{(2)} = (1,1,0)t, \text{ and } X^{(3)} = (1,1,1)^t.$$

Using Gram-Schmidt Process to determine a set of orthogonal vectors $V^{(1)}$, $V^{(2)}$, $V^{(3)}$ from $X^{(1)}$, $X^{(2)}$, $X^{(3)}$ then $V^{(2)} =$

(a) $(0,1,0)^t$

(b) $(0,0,1)^t$

(c) $(1,0,0)^t$

(d) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)^t$

(e) $\left(\frac{1}{2}, \frac{\sqrt{1}}{2}, 0\right)^t$

6. Given the following data:

| $i$ | $x_i$ | $y_i$ |
|---|---|---|
| 1 | 1 | 5.1 |
| 2 | 1.25 | 5.79 |
| 3 | 1.5 | 6.53 |

Construct the least squares approximation of the form $P(x) = be^{ax}$, then $P(1.5) \approx$

(a) 6.5368

(b) 7.5368

(c) 5.5368

(d) 4.5573

(e) 5.5666

7. If the Euler's Method is used to approximate the initial-value problem

$$y' = e^{t-y},\ 0 \leq t \leq 1,\ y(0) = 1, \text{ with } h = 0.5, \text{ then } w_2 \approx$$

(a) 1.4363
(b) 1.5243
(c) 2.3363
(d) 1.2343
(e) 1

8. Given the following data $f(1.4) = 10.3$, $f(1.5) = 12.3$, $f(1.6) = 15.3$, $f(1.8) = 20$. Using the three points formula to approximate $f'(1.6)$

(a) 35
(b) 34
(c) 36
(d) 33
(e) 37

9. Using the Composite Simpson's rule with $n = 4$ to approximate $\int_0^2 x\cos^2 x\,dx$, $\int_0^2 x\cos^2 x\,dx \approx$

(a) 0.4168
(b) 0.5178
(c) 0.61670
(d) 0.8168
(e) 0.32178

10. If the linear Finite-Difference Method is used to approximate the solutions of the boundary value problem

$$y'' = 4y,\ 0 \le x \le 1,\ y(0) = 1,\ y(1) = e^{(-2)}$$

with $h = \frac{1}{4}$, then $y\left(\frac{1}{2}\right) \approx$

(a) 0.3707
(b) 0.4370
(c) 0.4717
(d) 0
(e) 1

11. Given the nonlinear system

$$\begin{aligned} x_1^2 + x_2 &= 37 \\ x_1 - x_2^2 &= 5 \\ x_1 + x_2 + x_3 &= 3 \end{aligned}$$

Using the Newton's method with $X^{(0)} = (0,0,0)^t$ then $X^{(2)} \approx$

(a) $[4.3509, 18.4912, -19.8421]^t$
(b) $[4.9509, 18.4912, -21.8421]^t$
(c) $[4.3509, 17.4912, -19.7821]^t$
(d) $[5.3509, 18.8912, -19.1432]^t$
(e) $[1, 1, 1, 1]$

12. Using Newton's Method to find a solution for $2x = \sin x + 1$, by taking $p_0 = 0$, then $p_2 =$

(a) 0.8914
(b) 0.7904
(c) 1.0101
(d) 0.6704
(e) 0.6604

13. Given the non linear system

$$x_2 x_1 + x_2 = 1$$
$$x_1 - x_2^2 = 3$$

Using the method of steepest Descent with $X^{(0)} = [0, 0]^t$ and $\alpha = 0.1$, the $X^{(2)} \approx$

(a) $[1.1152, 0.2240]^t$
(b) $[1.1153, 0.3241]^t$
(c) $[1.2153, 0.3451]^t$
(d) $[1.3152, 0.4321]^t$
(e) $[1, 1]^t$

14. Let $f(0) = 1$, $f(1) = 0$, and $f(2) = -2$. If the Lagrange interpolating polynomial of degree two is used, then $f(1.5) \approx$

(a) $-0.875$
(b) $-0.987$
(c) $-0.785$
(d) $-1.175$
(e) $-1$

15. A clamped cubic spline $S$ for a function $f$ is defined by

$$S(x) = \begin{cases} S_0(x) = 1 + \beta x + 2x^2 - 2x^3, & \text{if } 0 \le x < 1 \\ S_1(x) = 1 + b(x-1) - 4(x-1)^2 + 7(x-1)^3, & \text{if } 1 \le x \le 2 \end{cases}$$

then $f'(0) + f'(2) =$

(a) 11
(b) 10
(c) 9
(d) 12
(e) 0

16. The row interchanges that are required to solve the following linear system using Gaussian Elimination with **partial pivoting**

$$\begin{aligned} 2x_1 - 3x_2 + 2x_3 &= 5 \\ -4x_1 + 2x_2 - 6x_3 &= 14 \\ 2x_1 + 2x_2 + 4x_3 &= 8 \end{aligned}$$

(a) interchange rows 1 and 2, then interchange rows 2 and 3
(b) interchange rows 1 and 2 only
(c) interchange rows 2 and 3 only
(d) interchange rows 1 and 3 only
(e) interchange rows 1 and 2, then interchange rows 1 and 3

17. If $A = LU$, where $L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$, $U$ is upper triangular form and $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \\ -1 & 3 & 2 \end{bmatrix}$, then $a + b + 2c =$

(a) 2
(b) 0
(c) 1
(d) 3
(e) 2.5

18. If $A = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 5 & -2 \\ -7 & 2 & 0 \end{bmatrix}$, then $\| A \|_\infty =$

(a) 9
(b) 4
(c) 3
(d) 2
(e) 0

19. Let $X^{(0)} = (0,0,0)$. If $X^{(2)}$ is the second iteration of the Jacobi method for the system

$$\begin{aligned} 10x_1 - x_2 &= 9 \\ -x_1 + 10x_2 - 2x_3 &= 7 \\ -2x_2 + 10x_3 &= 6, \end{aligned}$$

then $x_2^{(2)} \approx$

(a) 0.91
(b) 0.99
(c) 1
(d) 0.98
(e) 0.95

20. Given the boundary-value problem

$$y'' = 4y - 4x,\ 0 \le x \le 1,\ y(0) = 0,\ y(1) = 2$$

Using the linear finite difference method with $h = \frac{1}{4}$, to find the matrix form $AW = b$, then the sum of second column of $A$ is equal to

(a) $\frac{1}{4}$
(b) $\frac{1}{2}$
(c) $\frac{3}{4}$
(d) 0
(e) 1