

King Fahd University of Petroleum and Minerals  
Department of Mathematics  
**Math 371**  
**Exam 1**  
**Semester 243**  
**July 02, 2025**  
**Net Time Allowed: 90 Minutes**

**MASTER VERSION**

1. If  $P_3(x)$  is the third Taylor polynomial for the function  $f(x) = (x - 1)\ln(x)$  about  $x_0 = 1$ , then the absolute error  $|f(0.5) - P_3(0.5)|$  is equal to,

- (a) 0.0341 \_\_\_\_\_(correct)
- (b)  $-0.0341$
- (c) 0.0231
- (d) 0.0134
- (e) 0.0431

2. Using three-digit rounding arithmetic to calculate  $\frac{\frac{13}{14} - \frac{6}{7}}{2e - 5.4}$ ,

- (a) 1.80 \_\_\_\_\_(correct)
- (b) 1.85
- (c) 1.94
- (d) 1.97
- (e) 0.82

3. Suppose  $p^*$  must approximate 900 with relative error at most  $10^{-4}$ , then the largest interval in which  $p^*$  must lie is,

- (a) (899.91, 900.09) \_\_\_\_\_(correct)
- (b) (899.19, 900.09)
- (c) (899.90, 900.19)
- (d) (899.94, 900.04)
- (e) (899.00, 900.00)

4. The bisection method converge to which of the zero of  $f(x) = (x + 2)(x + 1)x(x - 1)^3(x - 2)$  in the interval  $[-3, 2.5]$ ,

- (a) 2 \_\_\_\_\_(correct)
- (b) -2
- (c) 0
- (d) 1
- (e) -1

5. Approximate solution of  $x^3 - x - 1 = 0$  on  $[1, 2]$  accurate to within  $10^{-2}$  with initial guess  $p_0 = 1$  and  $g(x) = (x + 1)^{\frac{1}{3}}$  is,

- (a) 1.32427 \_\_\_\_\_(correct)
- (b) 1.73242
- (c) 1.52427
- (d) 4 1.6724
- (e) 1.52837

6. The function  $g(x) = \cos(x)$  has a unique fixed point in the interval  $[0, 1]$ . Using  $p_0 = 0$ , the minimum number of iterations required to achieve  $10^{-5}$  accuracy by fixed point method is,

- (a) 67 \_\_\_\_\_(correct)
- (b) 76
- (c) 60
- (d) 55
- (e) 35

7. For which initial guess  $p_0$ , the Newton's method fails to approximate a root of  $x^3 - 2x^2 - 5 = 0$ ,

- (a)  $p_0 = \frac{4}{3}$  \_\_\_\_\_(correct)
- (b)  $p_0 = \frac{2}{3}$
- (c)  $p_0 = \frac{4}{5}$
- (d)  $p_0 = \frac{5}{3}$
- (e)  $p_0 = 4$

8. Let  $f(x) = -\cos(x) - x^3$ . If secant method is used with  $p_0 = -1$  and  $p_1 = 0$ , then  $p_3$  is equal to,

- (a)  $-1.25208$  \_\_\_\_\_(correct)
- (b)  $1.55208$
- (c)  $-1.45208$
- (d)  $1.35504$
- (e)  $-1.15310$

9. Let  $x_0 = 0$ ,  $x_1 = \frac{1}{6}$ , and  $x_2 = \frac{1}{2}$ . If second degree Lagrange polynomial  $P_2(x)$  is used to approximate  $f(x) = \sin(\pi x)$ . Then  $P_2(0.25)$  is,

- (a) 0.6875 \_\_\_\_\_(correct)
- (b) 0.4875
- (c) 0.1875
- (d) 0.7875
- (e) 0.8751

10. If the Natural cubic spline  $S(x) = \begin{cases} S_0(x) = a_0 + b_0(x-1) + c_0(x-1)^2 + d_0(x-1)^3, & \text{if } 1 \leq x < 2 \\ S_1(x) = a_1 + b_1(x-2) + c_1(x-2)^2 + d_1(x-2)^3, & \text{if } 2 \leq x \leq 3 \end{cases}$  defined on the interval  $[1, 3]$  interpolate the data  $(1, 1)$ ,  $(2, 1)$ ,  $(3, 0)$  then  $a_0 + a_1 + c_0 + c_1 + b_1 + d_1$  is equal to,

- (a) 1.75 \_\_\_\_\_(correct)
- (b) 1.50
- (c) 1.55
- (d) 2.45
- (e) 2.25

11. Let  $f(2.1) = 3.4321$ ,  $f(2.3) = 4.3210$  and  $f(2.5) = 5.3210$ . Using most accurate 3-points formulas, the sum  $f'(2.1) + f'(2.3)$  is equal to,

- (a) 8.8889 \_\_\_\_\_(correct)
- (b) 8.4587
- (c) 7.9880
- (d) 8.0998
- (e) 8.2682

12. Let  $f(1.29) = 13.7818$ ,  $f(1.3) = 14.0428$  and  $f(1.31) = 14.3074$ . The approximate value of  $f''(1.3)$  using  $h = 0.01$  is equal to,

- (a) 36.000 \_\_\_\_\_(correct)
- (b) 38.0012
- (c) 34.0005
- (d) 37.0102
- (e) 35.4090

13. Approximate value of the integral  $\int_0^2 e^{2x} \cos(3x) dx$  using the composite Trapezoidal rule with  $n = 4$  is,

- (a) 7.6775 \_\_\_\_\_(correct)
- (b) 4.6775
- (c) 9.6757
- (d) 5.7765
- (e) 3.2314

14. The smallest value of  $n$  required to approximate the integral  $\int_0^\pi \sin(x) dx$  by composite Simpson's rule with approximation error less than 0.00002 is,

- (a) 18 \_\_\_\_\_(correct)
- (b) 16
- (c) 15
- (d) 25
- (e) 9