King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 371

Exam 2

Semester 243

July 17, 2025

Net Time Allowed: 90 Minutes

MASTER VERSION

1. Which of the functions does not satisfy the Lipschitz condition on the domain D?

(a) $f(t,y) = t\sqrt{y-1} + y\sin(t)$ on $D = \{(t,y) \mid 0 \le t \le \pi \text{ and } 1 \le y \le 2\}$ _(correct)
(b) $f(t,y) = \frac{2+3y}{1+2t}$ on $D = \{(t,y) \mid 0 \le t \le 1 \text{ and } -\infty < y < \infty\}$
(c) $f(t,y) = \pi yt + te^y$ on $D = \{(t,y) \mid 1 \le t \le 2 \text{ and } -2 \le y \le 2\}$
(d) $f(t,y) = -\ln(y) + t\sqrt{y}$ on $D = \{(t,y) \mid 2 \le t \le 3 \text{ and } 2 \le y \le 3\}$
(e) $f(t,y) = e^t y + t\cos(yt)$ on $D = \{(t,y) \mid 0 \le t \le 2 \text{ and } -\infty < y < \infty\}$

2. The Euler method is used to approximate the solution of the initial value problem $y' = te^{3t} - 2y, \quad y(0) = 0, \quad 0 \le t \le 1$, with $h = 0.25$. If $y(t) = \frac{1}{5}te^{3t} - \frac{1}{25}e^{3t} + \frac{1}{25}e^{-2t}$ is the exact solution, then the absolute error $|y(t_2) - w_2|$ is equal to,

(a) 0.1513 ____________________(correct)
(b) 0.2543
(c) 0.3524
(d) 0.0552
(e) 0.1951

3. Consider the initial value problem $y' = 1 + \frac{y}{t}$, $y(1) = 2$, $1 \le t \le 2$, with $h = 0.25$. If $y(t) = t\ln t + 2t$ is the exact solution, then the error bound for $|y(t_2) - w_2|$ is equal to,
Note: this is not the absolute error.

(a) 0.0811 ____ (correct)
(b) 0.0899
(c) 0.0181
(d) 0.0118
(e) 0.0281

4. If the Runge-Kutta method of order 4 is used to approximate the solution of the initial value problem $y' = 1 + (t - y)^2$, $y(2) = 1$, $2 \le t \le 3$, with $h = 0.5$, then w_1 is equal to,

(a) 1.8333 ____ (correct)
(b) 1.3735
(c) 1.2796
(d) 1.9950
(e) 1.6537

5. The linear system

$$\begin{aligned} x_1 + 2x_2 + 4x_3 &= 11, \\ 2x_1 + 5x_2 + 2x_3 &= 3, \\ 4x_1 - x_2 + x_3 &= 8 \end{aligned}$$

is solved to find $x_1,\ x_2,\ x_3$. Then $x_1 + x_2 + x_3$ is equal to,

(a) 3 ______________________________ (correct)
(b) 5
(c) -5
(d) -3
(e) -2

6. The value of α so that the linear system,

$$\begin{aligned} x_1 - x_2 + \alpha x_3 &= 3 \\ -x_1 + 2x_2 - \alpha x_3 &= -2 \\ \alpha x_1 + x_2 + 4x_3 &= 9 \end{aligned}$$

has infinitely many solutions is,

(a) 2 ______________________________ (correct)
(b) -2
(c) 4
(d) -4
(e) 3

7. If Gaussian elimination with partial pivoting and three-digit rounding arithmetic is used to solve the following linear systems,

$$\begin{aligned} 0.03x_1 + 58.9x_2 &= 59.2 \\ 5.31x_1 - 6.10x_2 &= 47.0 \end{aligned}$$

then $x_1 + x_2$ is equal to,

(a) 11 ________________________________ (correct)
(b) 10
(c) 9
(d) 12
(e) 15

8. Consider the linear system,

$$\begin{aligned} 2x_1 + x_2 - x_3 &= 1 \\ -4x_1 + 4x_2 + 4x_3 &= 2 \\ 6x_1 + 3x_2 + 2x_3 &= 5 \end{aligned}$$

If the coefficient matrix A is factorized as LU and $LY = B$ is solved for $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$.

Then the sum of $y_1,\ y_2,\ y_3$ is equal to,

(a) 7 ________________________________ (correct)
(b) 6
(c) 5
(d) 8
(e) 9

9. If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ and $P_2P_1A = \begin{bmatrix} a_{31} & a_{32} & a_{33} \\ a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$, then the permutation matrices P_1 and P_2 are,

(a) $P_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ and $P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ ________________________________ (correct)

(b) $P_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(c) $P_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ and $P_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(d) $P_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ and $P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(e) $P_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $P_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

10. If $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & -4 & 6 \\ 1 & 3 & 5 \end{bmatrix}$ and $b = \begin{bmatrix} 8 \\ 6 \\ 0 \end{bmatrix}$, then $\|A\|_\infty + \|b\|_2$ is equal to,

(a) 22 ________________________________ (correct)

(b) 25

(c) 30

(d) 15

(e) 18

11. If $A = \begin{bmatrix} 2 & 2 \\ -1 & 2 \end{bmatrix}$, then $\|A\|_2$ is equal to,

(a) 3 ____ (correct)
(b) 4
(c) 5
(d) 6
(e) 9

12. The matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$ has eigenvalues $5, 1, 1$. Which one of these is an eigenvector of A,

(a) $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ ____ (correct)

(b) $\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

(e) $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

13. If the following system is solved using the Jacobi method with initial guess $\mathbf{x}^{(\mathbf{0})} = [0, 0, 0]$

$$\begin{aligned} 3x_1 - x_2 + x_3 &= 1 \\ 3x_1 + 6x_2 + 2x_3 &= 0 \\ 3x_1 + 3x_2 + 7x_3 &= 4 \end{aligned}$$

then $x_1^{(2)} + x_2^{(2)} + x_3^{(2)}$ is equal to,

(a) $\frac{3}{14}$ ——————————————————————————(correct)
(b) $\frac{6}{7}$
(c) $\frac{7}{16}$
(d) $\frac{5}{14}$
(e) $\frac{5}{21}$

14. If the following system is solved using the Gauss-Siedel method with initial guess $\mathbf{x}^{(\mathbf{0})} = [1, 1, 1]$

$$\begin{aligned} 4x_1 + x_2 - x_3 &= 5 \\ -x_1 + 3x_2 + x_3 &= -4 \\ 2x_1 + 2x_2 + 5x_3 &= 1 \end{aligned}$$

then $x_1^{(1)} + x_2^{(1)} + x_3^{(1)}$ is equal to,

(a) $\frac{1}{5}$ ——————————————————————————(correct)
(b) $\frac{4}{5}$
(c) $\frac{11}{4}$
(d) $\frac{5}{6}$
(e) $\frac{6}{5}$