King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 371

Major Exam I

251

September 28 , 2025

Net Time Allowed: 90 Minutes

MASTER VERSION

1. If the third **Taylor polynomial** $P_3(x)$ approximates $f(x) = (x-1)\ln(x)$ about $x_0 = 1$, then $P_3(0.8) =$

   (a) 0.0440 ____ (correct)

   (b) 0.0400

   (c) 0.0445

   (d) 0.0420

   (e) 0.0450

2. If the third **Taylor polynomial** $P_3(x)$ approximates $f(x) = e^{2x}\ \cos(2x)$ about $x_0 = 0$, then the least upper bound for $|f(x) - P_3(x)|$ on the interval $[-0.1, 0.1]$ is equal to

   (a) 3.2571×10^{-4} ____ (correct)

   (b) 3.6667×10^{-4}

   (c) 2.7513×10^{-4}

   (d) 3.3334×10^{-3}

   (e) 3.6667×10^{-3}

3. If $f(x) = \dfrac{5\ x + 8\ e}{x+3}$, then using **three-digit chopping** arithmetic $f(\pi)$ is equal to

(a) 6.07 ________________ (correct)
(b) 6.09
(c) 7.08
(d) 7.03
(e) 6.77

4. Suppose p^* must approximate $p = 900$ to **three-significant digits**. Then the largest interval in which p^* must lie is

(a) [895.5, 904.5] ________________ (correct)
(b) [897, 903]
(c) (895.5, 904.5)
(d) (897, 903)
(e) [897.5, 903.5]

5. Using the **Bisection** method to approximate the zero of $f(x) = x^3 + 4x^2 - 10$ on [1, 2], then $p_4 =$

   (a) 1.3125 ________ (correct)

   (b) 1.2577

   (c) 1.3157

   (d) 1.3515

   (e) 1.1253

6. The function $g(x) = \pi + 0.5\sin(\frac{x}{2})$ has a unique fixed point on $[0, 2\pi]$. Use $p_0 = \frac{\pi}{2}$ and the **error inequality** to estimate the minimum number of iterations required to achieve 10^{-8} accuracy by **fixed point iteration** method

   (a) 14 ________ (correct)

   (b) 16

   (c) 12

   (d) 10

   (e) 18

7. The equation $e^x - 3x^2 = 0$ has two positive solutions. Use **Newton's method** with $p_0 = 4$ to find solution accurate to within 10^{-6}:

   (a) 3.733079 ____ (correct)
   (b) 3.735312
   (c) 3.784361
   (d) 3.784578
   (e) 3.788234

8. Let $f(x) = x^2 - 6$. If the root of f is approximated by the **Secant method** with $p_0 = 3$ and $p_1 = 2$, then $p_3 =$

   (a) 2.4545 ____ (correct)
   (b) 2.4449
   (c) 2.4000
   (d) 2.4995
   (e) 2.4949

9. Let $P_2(x)$ be the **second Lagrange polynomial** for $f(x) = \tan(1 + x)$ using the nodes $x_0 = 0$, $x_1 = 0.6$, and $x_2 = 0.9$. Then $P_2(0.45) =$

   (a) -37.5851 ________________________________ (correct)

   (b) 8.2380

   (c) 0.0906

   (d) 12.9012

   (e) 38.8953

10. If $S(x)$ is a **natural cubic spline** that interpolates a function $f(x)$ at the points $(1, 1), (2, 6)$ and $(4, 4)$, then $S(3) =$

    (a) 13/2 ________________________________ (correct)

    (b) 5

    (c) 6

    (d) 29/4

    (e) 11/2

11. **A clamped cubic spline** S for a function f is defined on $[1, 3]$ by

$$\begin{cases} S_0(x) = 1 + b(x-1) + 3(x-1)^2 + a(x-1)^3, & 1 \le x < 2, \\ S_1(x) = 4 + d(x-2)^2 + 5(x-2)^3, & 2 \le x \le 3. \end{cases}$$

Given that $f'(1) = f'(3)$, then $a^2 + b^2 + d^2 =$

(a) 54 ______________________________ (correct)

(b) 13

(c) 34

(d) 49

(e) 64

12. Consider the data in the given table. Using **the most accurate** three-point formula, $f'(8.6) \approx$

x	8.3	8.4	8.5	8.6	8.8
$f(x)$	17.56492	17.87714	18.19056	18.50515	19.13781

(a) 3.151808 ______________________________ (correct)

(b) 2.351401

(c) 3.198091

(d) 2.351808

(e) 2.198091

13. Use the **composite Simpson's rule** with $n = 6$ to approximate the integral

$$\int_0^{12} \frac{x}{x^2+4}\,dx \approx$$

(a) 1.8007 ____ (correct)

(b) 1.8557

(c) 1.5777

(d) 1.8054

(e) 1.5007

14. By using the error term in **Composite Trapezoidal rule**, the smallest value of n required to approximate

$$\int_0^{\pi} \sin(x)dx \quad \text{within} \quad 10^{-4} \quad \text{is}$$

(a) 161 ____ (correct)

(b) 360

(c) 257

(d) 20

(e) 38