King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 371

Major Exam II

251

November 09 , 2025

Net Time Allowed: 90 Minutes

MASTER VERSION

1. Consider the differential equation:

$$(2+t^2)y' = (2+t^2)^2 y + \frac{t}{7}, \quad 0 \le t \le 2.$$

Given that $y' = f(t, y)$, then

(a) f satisfies Lipschitz condition with Lipschitz constant 6. ________________(correct)
(b) f satisfies Lipschitz condition with Lipschitz constant 3/2.
(c) f satisfies Lipschitz condition with Lipschitz constant 2.
(d) f satisfies Lipschitz condition with Lipschitz constant 1/7.
(e) f does not satisfy Lipschitz condition.

2. Consider the initial value problem:

$$y' = 1 + \frac{y}{t}, \quad y(1) = 2, \quad 1 \le t \le 6,$$

with exact solution $y(t) = t\ln(t) + 2t$. If the **Euler's method** is used with $h = 0.2$ to approximate the solution, then the upper bound for $|y(2.2) - w_6|$ is

(a) 0.232012 ____(correct)
(b) 0.203871
(c) 0.214023
(d) 0.230439
(e) 0.229271

3. Consider the initial value problem:

$$y' = 1 + (t - y)^2, \quad 2 \le t \le 3, \quad y(2) = 1, \text{with } h = 0.5,$$

with the exact solution $y(t) = t + \dfrac{1}{1-t}$. If the **midpoint method** is used with $h = 0.5$ to approximate the solution, then the absolute error between the exact and approximate solutions at $t = 3$ is:

(a) 0.044936 ________________________________ (correct)
(b) 2.445093
(c) 0.054938
(d) 0.055936
(e) 2.059361

4. Consider the initial value problem:

$$y' = y - t + 3, \quad 0 \le t \le 4, \quad y(0) = 1,$$

If the **Euler's method** is used with $h = 0.5$ to approximate the solution, then $y(2.00) \approx$

(a) 15.1875 ________________________________ (correct)
(b) 23.5761
(c) 9.9657
(d) 14.4422
(e) 15.40378

5. Consider the linear system

$$\begin{cases} x_1 + 2x_2 - x_3 = 1, \\ x_1 + 3x_2 + x_3 = 2, \\ x_1 + 2x_2 + (t+1)x_3 = s^2, \end{cases}$$

Which of the following is **FALSE** ?

(a) The system has no solution if $t = 2$ and $s = -1$ ________________ (correct)

(b) The system has infinitly many solutions if $t = -2$ and $s = 1$

(c) The system has no solution if $t = -2$ and $s = 2$

(d) The system has a unique solution if $t = 1$ and $s = 1$

(e) The system has a unique solution if $t = 2$ and $s = 3$

6. If the Gaussian elimination with **partial pivoting** and three digit **chopping** arithmetic is used to solve the linear system $\begin{cases} 0.1x_1 + 0.2x_2 = 2.3, \\ 0.2x_1 + 0.8x_2 = 9.99, \end{cases}$ then $x_1 + x_2 =$

(a) 9.85 ________________ (correct)

(b) 9.54

(c) 9.51

(d) 9.68

(e) 9.73

7. The row interchanges required to solve the linear system,

$$\begin{cases} 2x_1 + 12x_2 + x_3 + 2x_4 = 17, \\ -4x_1 + 2x_2 - 6x_3 + 2x_4 = -6, \\ 4x_1 + x_2 + 8x_3 + 4x_4 = 17 \\ 8x_1 + 8x_2 + 4x_3 + 4x_4 = 24 \end{cases}$$

using Gaussian elimination with **partial pivoting** is/are

(a) Interchange rows 1 and 4, then interchange rows 2 and 4. ______________(correct)
(b) Interchange rows 1 and 2 only.
(c) Interchange rows 1 and 4, then interchange rows 2 and 3.
(d) Interchange rows 1 and 4, then interchange rows 3 and 4.
(e) Interchange rows 1 and 4 only.

8. If the vector $x = [x_1, x_2, x_3, x_4]^t$ is the solution of the linear system

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 & 3 \\ 0 & 3 & 2 & -1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix},$$

then $x_1 \quad =$
(Hint: The coefficient matrix A is written as LU)

(a) $\frac{5}{12}$ ____(correct)
(b) $-\frac{1}{3}$
(c) $\frac{1}{2}$
(d) 0
(e) 1

9. Consider the initial value problem:

$$y' = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5,$$

If **the Runge-Kutta of order four** is used with $h = 0.5$ to approximate the solution, then $y(0.5) \approx$

(a) 1.42513 ________________________________ (correct)
(b) 1.93108
(c) 1.82049
(d) 1.44223
(e) 1.40378

10. If the following system is solved using **Jacobi iterative method**,

$$\begin{cases} 10x_1 + 5x_2 = 20, \\ 5x_1 + 10x_2 - 4x_3 = 30, \\ -4x_2 + 8x_3 - x_4 = -16, \\ -x_3 + 5x_4 = -10 \end{cases}$$

with $\mathbf{x}^{(0)} = (0, 0, 0, 0)^t$, then $|| \, x^{(2)} \, ||_\infty \; =$

(a) 2.4 ________________________________ (correct)
(b) 1.2
(c) 4.9
(d) 3
(e) 2.6

11. If the following system is solved using **Gauss-Seidel iterative method**,

$$\begin{cases} 3x_1 - x_2 + x_3 = 3, \\ 3x_1 + 6x_2 + 2x_3 = 9, \\ 3x_1 + 3x_2 + 7x_3 = 13, \end{cases}$$

with $\mathbf{x}^{(0)} = (3, 0, 0)^t$. Then $x_3^{(2)} =$

(a) 1.14286 ________________________________ (correct)

(b) 0.42849

(c) 2.42686

(d) 3.28136

(e) 1.04206

12. If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & -2 \\ 0 & 1 & 0 \end{bmatrix}$, then **the spectral radius** $\rho(A)$ of the matrix A is equal to

(a) $\sqrt{2}$ ________________________________ (correct)

(b) $\sqrt{3}$

(c) 1

(d) 2

(e) 3

13. What is the output of the following **MATLAB** code?

```
clear;
A=[2 1 1 1;-8 2 0 2;-5 0 -4 4;2 -3 1 1];
B = A(2:4,1:3);
C = A(2:3,2:3);
norm(B,inf) + norm(C)
```

(a) 14 ____ (correct)

(b) 10

(c) 12.5

(d) 13.5

(e) 4

14. If the output of the following **MATLAB** code

```
clear
A=[0 1 1 2;0 1 1 -1;1 2 -1 3;1 1 2 0];
[W,U]=lu(A)
```

is

```
W =
     0     1     0     1
     0     1     0     0
     1     0     0     0
     1    -1     1     0
U =
     1     2    -1     3
     0     1     1    -1
     0     0     4    -4
     0     0     0     3
```

and the matrix A can be factored as $A = (P^t\ L)\ U = W\ U$,then **the permutation matrix** P is equal to

(a) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ ____ (correct)

(b) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$