King Fahd University of Petroleum and Minerals
Department of Mathematics

**Math 371**
**Major Exam II**
**251**
**November 9, 2025**

# EXAM COVER

**Number of versions: 8**
**Number of questions: 14**

King Fahd University of Petroleum and Minerals
Department of Mathematics

**Math 371**

**Major Exam II**

**251**

**November 9, 2025**

**Net Time Allowed: 90 Minutes**

# MASTER VERSION

1. Which one of the following functions does not satisfy a Lipschitz condition on the domain $D$

   (a) $f(t,y) = y\ln y + t$ on $D = \{(t,y)| \quad 0 \le t \le 1, \quad 0 \le y \le 1\}$ ____________(correct)
   (b) $f(t,y) = (t^2+1)y^3$ on $D = \{(t,y)| \quad -1 \le t \le 1, \quad -1 \le y \le 1\}$
   (c) $f(t,y) = e^t \sin y$ on $D = \{(t,y)| \quad 0 \le t \le 2, \quad -\pi \le y \le \pi\}$
   (d) $f(t,y) = \dfrac{y}{1+t^2}$ on $D = \{(t,y)| \quad 0 \le t \le 3, \quad -2 \le y \le 2\}$
   (e) $f(t,y) = ye^{y^2} + t^3$ on $D = \{(t,y)| \quad -1 \le t \le 1, \quad -1 \le y \le 1\}$

2. Consider the initial value problem

$$y' = \tfrac{1}{2}y + t, \qquad 0 \le t \le 1, \qquad h = 0.2, \qquad y(0) = 1,$$

   with exact solution $y(t) = 5e^{t/2} - 2t - 4$. If Euler's method is used to approximate the solution, then the least bound for $|y(1) - w_5|$ is

   (a) 0.267390 ____________(correct)
   (b) 0.714785
   (c) 0.506500
   (d) 0.419500
   (e) 0.801250

3. Consider the initial value problem

$$y' = te^{3t} - 2y, \qquad 0 \le t \le 1, \qquad y(0) = 0, \qquad h = 0.5,$$

with exact solution

$$y(t) = \frac{1}{5}te^{3t} - \frac{1}{25}e^{3t} + \frac{1}{25}e^{-2t}.$$

If the midpoint method is used to approximate the solution, then the absolute error at $t = 1$ is

(a) 0.0891 ________________________________________ (correct)
(b) 0.0972
(c) 0.0792
(d) 0.0642
(e) 0.0312

4. Consider the initial value problem

$$y' = y - t^2 + 1, \qquad 0 \le t \le 2, \qquad y(0) = 0.5, \qquad h = 0.1.$$

If the fourth–order Runge–Kutta method is used to approximate the solution, then $y(0.2) \approx$

(a) 0.8292983 ________________________________________ (correct)
(b) 1.2140869
(c) 1.0150701
(d) 1.2056345
(e) 0.6574145

5. Use the Gaussian Elimination Algorithm (exact arithmetic, *no rounding*) to solve

$$\begin{cases} x_1 - x_2 + 3x_3 = 2, \\ 3x_1 - 3x_2 + x_3 = -1, \\ x_1 + x_2 \qquad = 3. \end{cases}$$

Then, $x_1 + x_2 + x_3 =$.

(a) 3.875 ______________________________ (correct)
(b) 3.5
(c) 4.000
(d) 3.625
(e) 3.750

6. Consider the linear system

$$\begin{aligned} -6x_1 - 12x_2 - x_3 &= -9, \\ 2x_1 + x_2 - x_3 &= 8, \\ 5x_1 + 12x_2 + x_3 &= 7. \end{aligned}$$

Let $R_i^{(k)}$ denote the $i$-th row of the augmented matrix at stage $k$ of Gaussian elimination and let $R_i^{(k)} \leftrightarrow R_j^{(k)}$ denote an interchange of the $i$-th and $j$-th rows. The row interchanges required to solve the system by *partial pivoting* are

(a) No row interchanges required in stages 1 and 2 ______________________________ (correct)
(b) Stage 1: $R_1^{(1)} \leftrightarrow R_3^{(1)}$, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$
(c) Stage 1: No row interchanges, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$
(d) Stage 1: $R_1^{(1)} \leftrightarrow R_3^{(1)}$, Stage 2: No row interchanges
(e) Stage 1: $R_1^{(1)} \leftrightarrow R_2^{(1)}$, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$

7. If

$$A = \begin{bmatrix} a & 1 & 0 \\ b & 2 & 3 \\ c & 0 & 1 \end{bmatrix}, \qquad \text{find the permutation matrix } P \text{ such that} \quad PA = \begin{bmatrix} c & 0 & 1 \\ b & 2 & 3 \\ a & 1 & 0 \end{bmatrix}.$$

The matrix $P$ is

(a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ ________________ (correct)

(b) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

8. Let

$$\mathbf{x} = \begin{pmatrix} 3 \\ -4 \\ 0 \\ \frac{3}{2} \end{pmatrix} \in \mathbb{R}^4.$$

Which statement about the norms of $\mathbf{x}$ is correct?

(a) $\|\mathbf{x}\|_2 > \|\mathbf{x}\|_\infty$ ________________ (correct)

(b) $\|\mathbf{x}\|_2 < \|\mathbf{x}\|_\infty$

(c) $\|\mathbf{x}\|_2 = \|\mathbf{x}\|_\infty$

(d) $\|\mathbf{x}\|_2 + \|\mathbf{x}\|_\infty < 9$

(e) $\|\mathbf{x}\|_2 - \|\mathbf{x}\|_\infty > 9$

9. Use the LU Factorization Algorithm to factor

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \\ -1 & 3 & 2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}}_{U}.$$

Compute $\ell_{32} + u_{33}$.

(a) 1 ______________________________ (correct)
(b) 2
(c) 1/2
(d) 0
(e) 3/2

10. Find the $\|A\|_\infty$ of the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

(a) 4 ______________________________ (correct)
(b) 12
(c) 2
(d) 1
(e) 11

11. For the linear system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix},$$

an approximate solution is $\tilde{\mathbf{x}} = \begin{bmatrix} -0.2 \\ -7.5 \\ 5.4 \end{bmatrix}$. Compute $\|A\tilde{\mathbf{x}} - \mathbf{b}\|_\infty$.

(a) 0.3 ______________________________ (correct)

(b) 0.4

(c) 0.2

(d) 0.5

(e) 0.1

12. Let

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

Find the spectral radius $\rho(A)$.

(a) 3 ______________________________ (correct)

(b) 2

(c) 1

(d) 4

(e) 5

13. Let $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ be the first and second iterates, respectively, of the Jacobi method with $\mathbf{x}^{(0)} = (1, 0, -1)^{\mathsf{T}}$ for the system

$$\begin{cases} 10x_1 - x_2 = 9, \\ -x_1 + 10x_2 - 2x_3 = 7, \\ -2x_2 + 10x_3 = 6. \end{cases}$$

Then $\left\|\mathbf{x}^{(2)} - \mathbf{x}^{(1)}\right\|_\infty =$

(a) 0.31 ______________________________ (correct)
(b) 0.06
(c) 0.12
(d) 0.21
(e) 0.14

14. Consider the system

$$\begin{cases} 4x_1 + x_2 - x_3 = 5, \\ -x_1 + 3x_2 + x_3 = -4, \\ 2x_1 + 2x_2 + 5x_3 = 1. \end{cases}$$

The first iterate of the Gauss-Seidel method in solving the system with initial guess $\mathbf{x}^{(0)} = (0, 0, 0)^{\mathsf{t}}$, is $\mathbf{x}^{(1)} =$

(a) $\left(1.250000000,\ -0.9166666667,\ 0.0666666667\right)^{\mathsf{t}}$ ______________ (correct)
(b) $\left(1.4958333333,\ -0.8569444444,\ -0.0555555556\right)^{\mathsf{t}}$
(c) $\left(0.9000000000,\ 0.7000000000,\ 0.6000000000\right)^{\mathsf{t}}$
(d) $\left(1.2500000000,\ -1.3333333333,\ 0.2000000000\right)^{\mathsf{t}}$
(e) $\left(1.4000000000,\ -0.9000000000,\ 0.0000000000\right)^{\mathsf{t}}$

King Fahd University of Petroleum and Minerals
Department of Mathematics

**CODE01** **CODE01**

# Math 371
# Major Exam II
# 251
# November 9, 2025
# Net Time Allowed: 90 Minutes

| Name | |
|---|---|

| ID | | Sec | |
|---|---|---|---|

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1. For the linear system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix},$$

an approximate solution is $\tilde{\mathbf{x}} = \begin{bmatrix} -0.2 \\ -7.5 \\ 5.4 \end{bmatrix}$. Compute $\|A\tilde{\mathbf{x}} - \mathbf{b}\|_\infty$.

(a) 0.2
(b) 0.5
(c) 0.1
(d) 0.4
(e) 0.3

2. Consider the initial value problem

$$y' = te^{3t} - 2y, \qquad 0 \le t \le 1, \qquad y(0) = 0, \qquad h = 0.5,$$

with exact solution

$$y(t) = \frac{1}{5}te^{3t} - \frac{1}{25}e^{3t} + \frac{1}{25}e^{-2t}.$$

If the midpoint method is used to approximate the solution, then the absolute error at $t = 1$ is

(a) 0.0642
(b) 0.0312
(c) 0.0792
(d) 0.0972
(e) 0.0891

3. Consider the linear system

$$\begin{aligned} -6x_1 - 12x_2 - x_3 &= -9, \\ 2x_1 + x_2 - x_3 &= 8, \\ 5x_1 + 12x_2 + x_3 &= 7. \end{aligned}$$

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(a) Stage 1: No row interchanges, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$
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(d) Stage 1: $R_1^{(1)} \leftrightarrow R_3^{(1)}$, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$
(e) Stage 1: $R_1^{(1)} \leftrightarrow R_3^{(1)}$, Stage 2: No row interchanges

4. Which one of the following functions does not satisfy a Lipschitz condition on the domain $D$

(a) $f(t,y) = y\ln y + t$ on $D = \{(t,y)| \quad 0 \le t \le 1, \quad 0 \le y \le 1\}$
(b) $f(t,y) = \dfrac{y}{1+t^2}$ on $D = \{(t,y)| \quad 0 \le t \le 3, \quad -2 \le y \le 2\}$
(c) $f(t,y) = e^t \sin y$ on $D = \{(t,y)| \quad 0 \le t \le 2, \quad -\pi \le y \le \pi\}$
(d) $f(t,y) = (t^2+1)y^3$ on $D = \{(t,y)| \quad -1 \le t \le 1, \quad -1 \le y \le 1\}$
(e) $f(t,y) = ye^{y^2} + t^3$ on $D = \{(t,y)| \quad -1 \le t \le 1, \quad -1 \le y \le 1\}$

5. Let

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

Find the spectral radius $\rho(A)$.

(a) 2
(b) 4
(c) 1
(d) 3
(e) 5

6. Consider the system

$$\begin{cases} 4x_1 + x_2 - x_3 = 5, \\ -x_1 + 3x_2 + x_3 = -4, \\ 2x_1 + 2x_2 + 5x_3 = 1. \end{cases}$$

The first iterate of the Gauss-Seidel method in solving the system with initial guess $\mathbf{x}^{(0)} = (0, 0, 0)^t$, is $\mathbf{x}^{(1)} =$

(a) $\left(0.9000000000,\ 0.7000000000,\ 0.6000000000\right)^t$
(b) $\left(1.4958333333,\ -0.8569444444,\ -0.0555555556\right)^t$
(c) $\left(1.250000000,\ -0.9166666667,\ 0.0666666667\right)^t$
(d) $\left(1.4000000000,\ -0.9000000000,\ 0.0000000000\right)^t$
(e) $\left(1.2500000000,\ -1.3333333333,\ 0.2000000000\right)^t$

7. Consider the initial value problem

$$y' = \frac{1}{2}y + t, \qquad 0 \le t \le 1, \qquad h = 0.2, \qquad y(0) = 1,$$

with exact solution $y(t) = 5e^{t/2} - 2t - 4$. If Euler's method is used to approximate the solution, then the least bound for $|y(1) - w_5|$ is

(a) 0.506500
(b) 0.419500
(c) 0.801250
(d) 0.267390
(e) 0.714785

8. Find the $\|A\|_\infty$ of the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

(a) 2
(b) 12
(c) 4
(d) 11
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9. Let $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ be the first and second iterates, respectively, of the Jacobi method with $\mathbf{x}^{(0)} = (1, 0, -1)^\mathsf{T}$ for the system

$$\begin{cases} 10x_1 - x_2 = 9, \\ -x_1 + 10x_2 - 2x_3 = 7, \\ -2x_2 + 10x_3 = 6. \end{cases}$$

Then $\left\|\mathbf{x}^{(2)} - \mathbf{x}^{(1)}\right\|_\infty =$

(a) 0.12
(b) 0.14
(c) 0.31
(d) 0.21
(e) 0.06

10. Let

$$\mathbf{x} = \begin{pmatrix} 3 \\ -4 \\ 0 \\ \frac{3}{2} \end{pmatrix} \in \mathbb{R}^4.$$

Which statement about the norms of $\mathbf{x}$ is correct?

(a) $\|\mathbf{x}\|_2 + \|\mathbf{x}\|_\infty < 9$
(b) $\|\mathbf{x}\|_2 < \|\mathbf{x}\|_\infty$
(c) $\|\mathbf{x}\|_2 > \|\mathbf{x}\|_\infty$
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11. Use the Gaussian Elimination Algorithm (exact arithmetic, *no rounding*) to solve

$$\begin{cases} x_1 - x_2 + 3x_3 = 2, \\ 3x_1 - 3x_2 + x_3 = -1, \\ x_1 + x_2 \qquad\quad = 3. \end{cases}$$

Then, $x_1 + x_2 + x_3 =$.

(a) 3.750
(b) 3.625
(c) 4.000
(d) 3.875
(e) 3.5

12. If

$$A = \begin{bmatrix} a & 1 & 0 \\ b & 2 & 3 \\ c & 0 & 1 \end{bmatrix}, \qquad \text{find the permutation matrix } P \text{ such that} \quad PA = \begin{bmatrix} c & 0 & 1 \\ b & 2 & 3 \\ a & 1 & 0 \end{bmatrix}.$$

The matrix $P$ is

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

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13. Consider the initial value problem

$$y' = y - t^2 + 1, \qquad 0 \le t \le 2, \qquad y(0) = 0.5, \qquad h = 0.1.$$

If the fourth–order Runge–Kutta method is used to approximate the solution, then $y(0.2) \approx$

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14. Use the LU Factorization Algorithm to factor

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \\ -1 & 3 & 2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}}_{U}.$$

Compute $\ell_{32} + u_{33}$.

(a) 3/2
(b) 0
(c) 1/2
(d) 2
(e) 1

King Fahd University of Petroleum and Minerals
Department of Mathematics

**CODE02** **CODE02**

**Math 371**
**Major Exam II**
**251**
**November 9, 2025**
**Net Time Allowed: 90 Minutes**

| **Name** | | | |
|---|---|---|---|
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Then, $x_1 + x_2 + x_3 =$.

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(b) 3.875
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6. Let

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Find the spectral radius $\rho(A)$.

(a) 4
(b) 1
(c) 5
(d) 2
(e) 3

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9. Find the $\|A\|_\infty$ of the matrix

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(a) 4
(b) 11
(c) 1
(d) 2
(e) 12

10. Consider the initial value problem

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with exact solution

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(a) Stage 1: $R_1^{(1)} \leftrightarrow R_3^{(1)}$, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$
(b) Stage 1: No row interchanges, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$
(c) Stage 1: $R_1^{(1)} \leftrightarrow R_3^{(1)}$, Stage 2: No row interchanges
(d) No row interchanges required in stages 1 and 2
(e) Stage 1: $R_1^{(1)} \leftrightarrow R_2^{(1)}$, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$

13. For the linear system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix},$$

an approximate solution is $\tilde{\mathbf{x}} = \begin{bmatrix} -0.2 \\ -7.5 \\ 5.4 \end{bmatrix}$. Compute $\|A\tilde{\mathbf{x}} - \mathbf{b}\|_\infty$.

(a) 0.1
(b) 0.3
(c) 0.4
(d) 0.5
(e) 0.2

14. Use the LU Factorization Algorithm to factor

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \\ -1 & 3 & 2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}}_{U}.$$

Compute $\ell_{32} + u_{33}$.

(a) 1/2
(b) 1
(c) 3/2
(d) 0
(e) 2

King Fahd University of Petroleum and Minerals
Department of Mathematics

**CODE03** **CODE03**

**Math 371**
**Major Exam II**
**251**
**November 9, 2025**
**Net Time Allowed: 90 Minutes**

| Name | |
|---|---|
| ID | | Sec | |

**Check that this exam has 14 questions.**

**Important Instructions:**

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3. Use a good eraser. DO NOT use the erasers attached to the pencil.
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6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. Consider the system

$$\begin{cases} 4x_1 + x_2 - x_3 = 5, \\ -x_1 + 3x_2 + x_3 = -4, \\ 2x_1 + 2x_2 + 5x_3 = 1. \end{cases}$$

The first iterate of the Gauss-Seidel method in solving the system with initial guess $\mathbf{x}^{(0)} = (0, 0, 0)^{\mathrm{t}}$, is $\mathbf{x}^{(1)} =$

(a) $(1.2500000000, -1.3333333333, 0.2000000000)^{\mathrm{t}}$
(b) $(1.4958333333, -0.8569444444, -0.0555555556)^{\mathrm{t}}$
(c) $(0.9000000000, 0.7000000000, 0.6000000000)^{\mathrm{t}}$
(d) $(1.250000000, -0.9166666667, 0.0666666667)^{\mathrm{t}}$
(e) $(1.4000000000, -0.9000000000, 0.0000000000)^{\mathrm{t}}$

2. If

$$A = \begin{bmatrix} a & 1 & 0 \\ b & 2 & 3 \\ c & 0 & 1 \end{bmatrix}, \qquad \text{find the permutation matrix } P \text{ such that} \quad PA = \begin{bmatrix} c & 0 & 1 \\ b & 2 & 3 \\ a & 1 & 0 \end{bmatrix}.$$

The matrix $P$ is

(a) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

3. Find the $\|A\|_\infty$ of the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

(a) 1
(b) 2
(c) 12
(d) 4
(e) 11

4. Use the Gaussian Elimination Algorithm (exact arithmetic, *no rounding*) to solve

$$\begin{cases} x_1 - x_2 + 3x_3 = 2, \\ 3x_1 - 3x_2 + x_3 = -1, \\ x_1 + x_2 \qquad = 3. \end{cases}$$

Then, $x_1 + x_2 + x_3 =$.

(a) 3.625
(b) 3.750
(c) 4.000
(d) 3.875
(e) 3.5

5. Consider the initial value problem

$$y' = y - t^2 + 1, \qquad 0 \le t \le 2, \qquad y(0) = 0.5, \qquad h = 0.1.$$

If the fourth–order Runge–Kutta method is used to approximate the solution, then $y(0.2) \approx$

(a) 1.2056345
(b) 0.6574145
(c) 1.2140869
(d) 0.8292983
(e) 1.0150701

6. Consider the initial value problem

$$y' = \tfrac{1}{2}y + t, \qquad 0 \le t \le 1, \qquad h = 0.2, \qquad y(0) = 1,$$

with exact solution $y(t) = 5e^{t/2} - 2t - 4$. If Euler's method is used to approximate the solution, then the least bound for $|y(1) - w_5|$ is

(a) 0.267390
(b) 0.419500
(c) 0.801250
(d) 0.506500
(e) 0.714785

7. Consider the linear system

$$-6x_1 - 12x_2 - x_3 = -9,$$
$$2x_1 + x_2 - x_3 = 8,$$
$$5x_1 + 12x_2 + x_3 = 7.$$

Let $R_i^{(k)}$ denote the $i$-th row of the augmented matrix at stage $k$ of Gaussian elimination and let $R_i^{(k)} \leftrightarrow R_j^{(k)}$ denote an interchange of the $i$-th and $j$-th rows. The row interchanges required to solve the system by *partial pivoting* are

(a) Stage 1: $R_1^{(1)} \leftrightarrow R_2^{(1)}$, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$
(b) Stage 1: $R_1^{(1)} \leftrightarrow R_3^{(1)}$, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$
(c) Stage 1: $R_1^{(1)} \leftrightarrow R_3^{(1)}$, Stage 2: No row interchanges
(d) Stage 1: No row interchanges, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$
(e) No row interchanges required in stages 1 and 2

8. Let $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ be the first and second iterates, respectively, of the Jacobi method with $\mathbf{x}^{(0)} = (1, 0, -1)^{\mathsf{T}}$ for the system

$$\begin{cases} 10x_1 - x_2 = 9, \\ -x_1 + 10x_2 - 2x_3 = 7, \\ -2x_2 + 10x_3 = 6. \end{cases}$$

Then $\left\|\mathbf{x}^{(2)} - \mathbf{x}^{(1)}\right\|_\infty =$

(a) 0.14
(b) 0.31
(c) 0.06
(d) 0.12
(e) 0.21

9. Let

$$\mathbf{x} = \begin{pmatrix} 3 \\ -4 \\ 0 \\ \frac{3}{2} \end{pmatrix} \in \mathbb{R}^4.$$

Which statement about the norms of $\mathbf{x}$ is correct?

(a) $\|\mathbf{x}\|_2 < \|\mathbf{x}\|_\infty$
(b) $\|\mathbf{x}\|_2 + \|\mathbf{x}\|_\infty < 9$
(c) $\|\mathbf{x}\|_2 - \|\mathbf{x}\|_\infty > 9$
(d) $\|\mathbf{x}\|_2 > \|\mathbf{x}\|_\infty$
(e) $\|\mathbf{x}\|_2 = \|\mathbf{x}\|_\infty$

10. Let

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

Find the spectral radius $\rho(A)$.

(a) 2
(b) 1
(c) 5
(d) 4
(e) 3

11. Consider the initial value problem

$$y' = te^{3t} - 2y, \qquad 0 \le t \le 1, \qquad y(0) = 0, \qquad h = 0.5,$$

with exact solution

$$y(t) = \frac{1}{5}te^{3t} - \frac{1}{25}e^{3t} + \frac{1}{25}e^{-2t}.$$

If the midpoint method is used to approximate the solution, then the absolute error at $t = 1$ is

(a) 0.0312
(b) 0.0891
(c) 0.0792
(d) 0.0642
(e) 0.0972

12. Which one of the following functions does not satisfy a Lipschitz condition on the domain $D$

(a) $f(t, y) = e^t \sin y$ on $D = \{(t, y)| \quad 0 \le t \le 2, \quad -\pi \le y \le \pi\}$
(b) $f(t, y) = (t^2 + 1)y^3$ on $D = \{(t, y)| \quad -1 \le t \le 1, \quad -1 \le y \le 1\}$
(c) $f(t, y) = \dfrac{y}{1 + t^2}$ on $D = \{(t, y)| \quad 0 \le t \le 3, \quad -2 \le y \le 2\}$
(d) $f(t, y) = y \ln y + t$ on $D = \{(t, y)| \quad 0 \le t \le 1, \quad 0 \le y \le 1\}$
(e) $f(t, y) = ye^{y^2} + t^3$ on $D = \{(t, y)| \quad -1 \le t \le 1, \quad -1 \le y \le 1\}$

13. Use the LU Factorization Algorithm to factor

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \\ -1 & 3 & 2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}}_{U}.$$

Compute $\ell_{32} + u_{33}$.

(a) 1
(b) 0
(c) 3/2
(d) 2
(e) 1/2

14. For the linear system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix},$$

an approximate solution is $\tilde{\mathbf{x}} = \begin{bmatrix} -0.2 \\ -7.5 \\ 5.4 \end{bmatrix}$. Compute $\|A\tilde{\mathbf{x}} - \mathbf{b}\|_\infty$.

(a) 0.2
(b) 0.5
(c) 0.4
(d) 0.1
(e) 0.3

King Fahd University of Petroleum and Minerals
Department of Mathematics

**CODE04** **CODE04**

**Math 371**
**Major Exam II**
**251**
**November 9, 2025**
**Net Time Allowed: 90 Minutes**

| **Name** | | | |
|---|---|---|---|
| **ID** | | **Sec** | |

**Check that this exam has 14 questions.**

## Important Instructions:

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7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. Let

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

Find the spectral radius $\rho(A)$.

(a) 1
(b) 3
(c) 5
(d) 2
(e) 4

2. Use the LU Factorization Algorithm to factor

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \\ -1 & 3 & 2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}}_{U}.$$

Compute $\ell_{32} + u_{33}$.

(a) 2
(b) 3/2
(c) 1/2
(d) 1
(e) 0

3. Consider the linear system

$$
\begin{aligned}
-6x_1 - 12x_2 - x_3 &= -9,\\
2x_1 + x_2 - x_3 &= 8,\\
5x_1 + 12x_2 + x_3 &= 7.
\end{aligned}
$$

Let $R_i^{(k)}$ denote the $i$-th row of the augmented matrix at stage $k$ of Gaussian elimination and let $R_i^{(k)} \leftrightarrow R_j^{(k)}$ denote an interchange of the $i$-th and $j$-th rows. The row interchanges required to solve the system by *partial pivoting* are

(a) Stage 1: $R_1^{(1)} \leftrightarrow R_3^{(1)}$, Stage 2: No row interchanges

(b) No row interchanges required in stages 1 and 2

(c) Stage 1: $R_1^{(1)} \leftrightarrow R_2^{(1)}$, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$

(d) Stage 1: $R_1^{(1)} \leftrightarrow R_3^{(1)}$, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$

(e) Stage 1: No row interchanges, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$

4. Which one of the following functions does not satisfy a Lipschitz condition on the domain $D$

(a) $f(t,y) = e^t \sin y$ on $D = \{(t,y) | \quad 0 \le t \le 2, \quad -\pi \le y \le \pi\}$

(b) $f(t,y) = ye^{y^2} + t^3$ on $D = \{(t,y) | \quad -1 \le t \le 1, \quad -1 \le y \le 1\}$

(c) $f(t,y) = (t^2+1)y^3$ on $D = \{(t,y) | \quad -1 \le t \le 1, \quad -1 \le y \le 1\}$

(d) $f(t,y) = y \ln y + t$ on $D = \{(t,y) | \quad 0 \le t \le 1, \quad 0 \le y \le 1\}$

(e) $f(t,y) = \dfrac{y}{1+t^2}$ on $D = \{(t,y) | \quad 0 \le t \le 3, \quad -2 \le y \le 2\}$

5. Find the $\|A\|_\infty$ of the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

(a) 1
(b) 2
(c) 12
(d) 11
(e) 4

6. Let

$$\mathbf{x} = \begin{pmatrix} 3 \\ -4 \\ 0 \\ \frac{3}{2} \end{pmatrix} \in \mathbb{R}^4.$$

Which statement about the norms of **x** is correct?

(a) $\|\mathbf{x}\|_2 - \|\mathbf{x}\|_\infty > 9$
(b) $\|\mathbf{x}\|_2 = \|\mathbf{x}\|_\infty$
(c) $\|\mathbf{x}\|_2 < \|\mathbf{x}\|_\infty$
(d) $\|\mathbf{x}\|_2 > \|\mathbf{x}\|_\infty$
(e) $\|\mathbf{x}\|_2 + \|\mathbf{x}\|_\infty < 9$

7. Let $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ be the first and second iterates, respectively, of the Jacobi method with $\mathbf{x}^{(0)} = (1, 0, -1)^{\mathsf{T}}$ for the system

$$\begin{cases} 10x_1 - x_2 = 9, \\ -x_1 + 10x_2 - 2x_3 = 7, \\ -2x_2 + 10x_3 = 6. \end{cases}$$

Then $\left\|\mathbf{x}^{(2)} - \mathbf{x}^{(1)}\right\|_\infty =$

(a) 0.31
(b) 0.14
(c) 0.21
(d) 0.06
(e) 0.12

8. For the linear system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix},$$

an approximate solution is $\tilde{\mathbf{x}} = \begin{bmatrix} -0.2 \\ -7.5 \\ 5.4 \end{bmatrix}$. Compute $\|A\tilde{\mathbf{x}} - \mathbf{b}\|_\infty$.

(a) 0.1
(b) 0.4
(c) 0.2
(d) 0.5
(e) 0.3

9. Use the Gaussian Elimination Algorithm (exact arithmetic, *no rounding*) to solve

$$\begin{cases} x_1 - x_2 + 3x_3 = 2, \\ 3x_1 - 3x_2 + x_3 = -1, \\ x_1 + x_2 \qquad\quad = 3. \end{cases}$$

Then, $x_1 + x_2 + x_3 =$.

(a) 3.750
(b) 3.625
(c) 3.5
(d) 4.000
(e) 3.875

10. Consider the initial value problem

$$y' = y - t^2 + 1, \qquad 0 \le t \le 2, \qquad y(0) = 0.5, \qquad h = 0.1.$$

If the fourth–order Runge–Kutta method is used to approximate the solution, then $y(0.2) \approx$

(a) 1.2140869
(b) 1.0150701
(c) 0.8292983
(d) 1.2056345
(e) 0.6574145

11. Consider the system

$$\begin{cases} 4x_1 + x_2 - x_3 = 5, \\ -x_1 + 3x_2 + x_3 = -4, \\ 2x_1 + 2x_2 + 5x_3 = 1. \end{cases}$$

The first iterate of the Gauss-Seidel method in solving the system with initial guess $\mathbf{x}^{(0)} = (0,0,0)^t$, is $\mathbf{x}^{(1)} =$

(a) $\left(1.250000000,\ -0.9166666667,\ 0.0666666667\right)^t$

(b) $\left(1.2500000000,\ -1.3333333333,\ 0.2000000000\right)^t$

(c) $\left(1.4958333333,\ -0.8569444444,\ -0.0555555556\right)^t$

(d) $\left(0.9000000000,\ 0.7000000000,\ 0.6000000000\right)^t$

(e) $\left(1.4000000000,\ -0.9000000000,\ 0.0000000000\right)^t$

12. If

$$A = \begin{bmatrix} a & 1 & 0 \\ b & 2 & 3 \\ c & 0 & 1 \end{bmatrix}, \qquad \text{find the permutation matrix } P \text{ such that} \quad PA = \begin{bmatrix} c & 0 & 1 \\ b & 2 & 3 \\ a & 1 & 0 \end{bmatrix}.$$

The matrix $P$ is

(a) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

13. Consider the initial value problem

$$y' = te^{3t} - 2y, \qquad 0 \le t \le 1, \qquad y(0) = 0, \qquad h = 0.5,$$

with exact solution

$$y(t) = \frac{1}{5}te^{3t} - \frac{1}{25}e^{3t} + \frac{1}{25}e^{-2t}.$$

If the midpoint method is used to approximate the solution, then the absolute error at $t = 1$ is

(a) 0.0891
(b) 0.0312
(c) 0.0642
(d) 0.0972
(e) 0.0792

14. Consider the initial value problem

$$y' = \tfrac{1}{2}y + t, \qquad 0 \le t \le 1, \qquad h = 0.2, \qquad y(0) = 1,$$

with exact solution $y(t) = 5e^{t/2} - 2t - 4$. If Euler's method is used to approximate the solution, then the least bound for $|y(1) - w_5|$ is

(a) 0.714785
(b) 0.419500
(c) 0.506500
(d) 0.801250
(e) 0.267390

King Fahd University of Petroleum and Minerals
Department of Mathematics

**CODE05** **CODE05**

**Math 371**
**Major Exam II**
**251**
**November 9, 2025**
**Net Time Allowed: 90 Minutes**

| **Name** | |
|---|---|

| **ID** | | **Sec** | |
|---|---|---|---|

**Check that this exam has 14 questions.**

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7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. Consider the initial value problem

$$y' = \tfrac{1}{2}y + t, \qquad 0 \le t \le 1, \qquad h = 0.2, \qquad y(0) = 1,$$

with exact solution $y(t) = 5e^{t/2} - 2t - 4$. If Euler's method is used to approximate the solution, then the least bound for $|y(1) - w_5|$ is

(a) 0.267390
(b) 0.714785
(c) 0.801250
(d) 0.506500
(e) 0.419500

2. Consider the linear system

$$\begin{aligned} -6x_1 - 12x_2 - x_3 &= -9, \\ 2x_1 + x_2 - x_3 &= 8, \\ 5x_1 + 12x_2 + x_3 &= 7. \end{aligned}$$

Let $R_i^{(k)}$ denote the $i$-th row of the augmented matrix at stage $k$ of Gaussian elimination and let $R_i^{(k)} \leftrightarrow R_j^{(k)}$ denote an interchange of the $i$-th and $j$-th rows. The row interchanges required to solve the system by *partial pivoting* are

(a) No row interchanges required in stages 1 and 2
(b) Stage 1: $R_1^{(1)} \leftrightarrow R_3^{(1)}$, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$
(c) Stage 1: $R_1^{(1)} \leftrightarrow R_3^{(1)}$, Stage 2: No row interchanges
(d) Stage 1: No row interchanges, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$
(e) Stage 1: $R_1^{(1)} \leftrightarrow R_2^{(1)}$, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$

3. Let

$$\mathbf{x} = \begin{pmatrix} 3 \\ -4 \\ 0 \\ \frac{3}{2} \end{pmatrix} \in \mathbb{R}^4.$$

Which statement about the norms of $\mathbf{x}$ is correct?

(a) $\|\mathbf{x}\|_2 - \|\mathbf{x}\|_\infty > 9$
(b) $\|\mathbf{x}\|_2 < \|\mathbf{x}\|_\infty$
(c) $\|\mathbf{x}\|_2 > \|\mathbf{x}\|_\infty$
(d) $\|\mathbf{x}\|_2 = \|\mathbf{x}\|_\infty$
(e) $\|\mathbf{x}\|_2 + \|\mathbf{x}\|_\infty < 9$

4. Use the Gaussian Elimination Algorithm (exact arithmetic, *no rounding*) to solve

$$\begin{cases} x_1 - x_2 + 3x_3 = 2, \\ 3x_1 - 3x_2 + x_3 = -1, \\ x_1 + x_2 \qquad = 3. \end{cases}$$

Then, $x_1 + x_2 + x_3 =$.

(a) 3.750
(b) 4.000
(c) 3.875
(d) 3.5
(e) 3.625

5. Consider the initial value problem

$$y' = te^{3t} - 2y, \qquad 0 \le t \le 1, \qquad y(0) = 0, \qquad h = 0.5,$$

with exact solution

$$y(t) = \frac{1}{5}te^{3t} - \frac{1}{25}e^{3t} + \frac{1}{25}e^{-2t}.$$

If the midpoint method is used to approximate the solution, then the absolute error at $t = 1$ is

(a) 0.0891
(b) 0.0972
(c) 0.0792
(d) 0.0642
(e) 0.0312

6. Let $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ be the first and second iterates, respectively, of the Jacobi method with $\mathbf{x}^{(0)} = (1, 0, -1)^\mathsf{T}$ for the system

$$\begin{cases} 10x_1 - x_2 = 9, \\ -x_1 + 10x_2 - 2x_3 = 7, \\ -2x_2 + 10x_3 = 6. \end{cases}$$

Then $\left\|\mathbf{x}^{(2)} - \mathbf{x}^{(1)}\right\|_\infty =$

(a) 0.21
(b) 0.12
(c) 0.06
(d) 0.31
(e) 0.14

7. Consider the system

$$\begin{cases} 4x_1 + x_2 - x_3 = 5, \\ -x_1 + 3x_2 + x_3 = -4, \\ 2x_1 + 2x_2 + 5x_3 = 1. \end{cases}$$

The first iterate of the Gauss-Seidel method in solving the system with initial guess $\mathbf{x}^{(0)} = (0,0,0)^t$, is $\mathbf{x}^{(1)} =$

(a) $\left(1.4958333333,\ -0.8569444444,\ -0.0555555556\right)^t$
(b) $\left(1.2500000000,\ -1.3333333333,\ 0.2000000000\right)^t$
(c) $\left(0.9000000000,\ 0.7000000000,\ 0.6000000000\right)^t$
(d) $\left(1.250000000,\ -0.9166666667,\ 0.0666666667\right)^t$
(e) $\left(1.4000000000,\ -0.9000000000,\ 0.0000000000\right)^t$

8. Which one of the following functions does not satisfy a Lipschitz condition on the domain $D$

(a) $f(t,y) = (t^2+1)y^3$ on $D = \{(t,y) | \quad -1 \le t \le 1, \quad -1 \le y \le 1\}$
(b) $f(t,y) = y \ln y + t$ on $D = \{(t,y) | \quad 0 \le t \le 1, \quad 0 \le y \le 1\}$
(c) $f(t,y) = ye^{y^2} + t^3$ on $D = \{(t,y) | \quad -1 \le t \le 1, \quad -1 \le y \le 1\}$
(d) $f(t,y) = \dfrac{y}{1+t^2}$ on $D = \{(t,y) | \quad 0 \le t \le 3, \quad -2 \le y \le 2\}$
(e) $f(t,y) = e^t \sin y$ on $D = \{(t,y) | \quad 0 \le t \le 2, \quad -\pi \le y \le \pi\}$

9. Use the LU Factorization Algorithm to factor

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \\ -1 & 3 & 2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}}_{U}.$$

Compute $\ell_{32} + u_{33}$.

(a) 0
(b) 3/2
(c) 1/2
(d) 2
(e) 1

10. Find the $\|A\|_\infty$ of the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

(a) 12
(b) 1
(c) 4
(d) 2
(e) 11

11. For the linear system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix},$$

an approximate solution is $\tilde{\mathbf{x}} = \begin{bmatrix} -0.2 \\ -7.5 \\ 5.4 \end{bmatrix}$. Compute $\|A\tilde{\mathbf{x}} - \mathbf{b}\|_\infty$.

(a) 0.2
(b) 0.1
(c) 0.4
(d) 0.3
(e) 0.5

12. Let

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

Find the spectral radius $\rho(A)$.

(a) 4
(b) 3
(c) 5
(d) 1
(e) 2

13. If

$$A = \begin{bmatrix} a & 1 & 0 \\ b & 2 & 3 \\ c & 0 & 1 \end{bmatrix}, \qquad \text{find the permutation matrix } P \text{ such that} \quad PA = \begin{bmatrix} c & 0 & 1 \\ b & 2 & 3 \\ a & 1 & 0 \end{bmatrix}.$$

The matrix $P$ is

(a) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

14. Consider the initial value problem

$$y' = y - t^2 + 1, \qquad 0 \le t \le 2, \qquad y(0) = 0.5, \qquad h = 0.1.$$

If the fourth–order Runge–Kutta method is used to approximate the solution, then $y(0.2) \approx$

(a) 1.0150701

(b) 1.2056345

(c) 0.6574145

(d) 0.8292983

(e) 1.2140869

King Fahd University of Petroleum and Minerals
Department of Mathematics

**CODE06** **CODE06**

# Math 371
## Major Exam II
## 251
## November 9, 2025
## Net Time Allowed: 90 Minutes

| Name | |
|---|---|
| ID | | Sec | |

**Check that this exam has 14 questions.**

## Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
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7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. Consider the initial value problem

$$y' = te^{3t} - 2y, \qquad 0 \le t \le 1, \qquad y(0) = 0, \qquad h = 0.5,$$

with exact solution

$$y(t) = \frac{1}{5}te^{3t} - \frac{1}{25}e^{3t} + \frac{1}{25}e^{-2t}.$$

If the midpoint method is used to approximate the solution, then the absolute error at $t = 1$ is

(a) 0.0972
(b) 0.0642
(c) 0.0312
(d) 0.0891
(e) 0.0792

2. For the linear system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix},$$

an approximate solution is $\tilde{\mathbf{x}} = \begin{bmatrix} -0.2 \\ -7.5 \\ 5.4 \end{bmatrix}$. Compute $\|A\tilde{\mathbf{x}} - \mathbf{b}\|_\infty$.

(a) 0.2
(b) 0.4
(c) 0.1
(d) 0.5
(e) 0.3

3. Consider the linear system

$$\begin{aligned} -6x_1 - 12x_2 - x_3 &= -9, \\ 2x_1 + x_2 - x_3 &= 8, \\ 5x_1 + 12x_2 + x_3 &= 7. \end{aligned}$$

Let $R_i^{(k)}$ denote the $i$-th row of the augmented matrix at stage $k$ of Gaussian elimination and let $R_i^{(k)} \leftrightarrow R_j^{(k)}$ denote an interchange of the $i$-th and $j$-th rows. The row interchanges required to solve the system by *partial pivoting* are

(a) Stage 1: $R_1^{(1)} \leftrightarrow R_3^{(1)}$, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$

(b) Stage 1: $R_1^{(1)} \leftrightarrow R_3^{(1)}$, Stage 2: No row interchanges

(c) Stage 1: $R_1^{(1)} \leftrightarrow R_2^{(1)}$, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$

(d) No row interchanges required in stages 1 and 2

(e) Stage 1: No row interchanges, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$

4. Use the LU Factorization Algorithm to factor

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \\ -1 & 3 & 2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}}_{U}.$$

Compute $\ell_{32} + u_{33}$.

(a) 0

(b) 3/2

(c) 1/2

(d) 2

(e) 1

5. Let

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

Find the spectral radius $\rho(A)$.

(a) 4
(b) 2
(c) 5
(d) 3
(e) 1

6. Consider the initial value problem

$$y' = \tfrac{1}{2}y + t, \qquad 0 \le t \le 1, \qquad h = 0.2, \qquad y(0) = 1,$$

with exact solution $y(t) = 5e^{t/2} - 2t - 4$. If Euler's method is used to approximate the solution, then the least bound for $|y(1) - w_5|$ is

(a) 0.801250
(b) 0.506500
(c) 0.419500
(d) 0.267390
(e) 0.714785

7. Let $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ be the first and second iterates, respectively, of the Jacobi method with $\mathbf{x}^{(0)} = (1, 0, -1)^\mathsf{T}$ for the system

$$\begin{cases} 10x_1 - x_2 = 9, \\ -x_1 + 10x_2 - 2x_3 = 7, \\ -2x_2 + 10x_3 = 6. \end{cases}$$

Then $\left\|\mathbf{x}^{(2)} - \mathbf{x}^{(1)}\right\|_\infty =$

(a) 0.12
(b) 0.31
(c) 0.14
(d) 0.21
(e) 0.06

8. Let

$$\mathbf{x} = \begin{pmatrix} 3 \\ -4 \\ 0 \\ \frac{3}{2} \end{pmatrix} \in \mathbb{R}^4.$$

Which statement about the norms of $\mathbf{x}$ is correct?

(a) $\|\mathbf{x}\|_2 = \|\mathbf{x}\|_\infty$
(b) $\|\mathbf{x}\|_2 - \|\mathbf{x}\|_\infty > 9$
(c) $\|\mathbf{x}\|_2 + \|\mathbf{x}\|_\infty < 9$
(d) $\|\mathbf{x}\|_2 > \|\mathbf{x}\|_\infty$
(e) $\|\mathbf{x}\|_2 < \|\mathbf{x}\|_\infty$

9. Which one of the following functions does not satisfy a Lipschitz condition on the domain $D$

(a) $f(t,y) = (t^2+1)y^3$ on $D = \{(t,y) | \quad -1 \le t \le 1, \quad -1 \le y \le 1\}$
(b) $f(t,y) = \dfrac{y}{1+t^2}$ on $D = \{(t,y) | \quad 0 \le t \le 3, \quad -2 \le y \le 2\}$
(c) $f(t,y) = e^t \sin y$ on $D = \{(t,y) | \quad 0 \le t \le 2, \quad -\pi \le y \le \pi\}$
(d) $f(t,y) = y \ln y + t$ on $D = \{(t,y) | \quad 0 \le t \le 1, \quad 0 \le y \le 1\}$
(e) $f(t,y) = ye^{y^2} + t^3$ on $D = \{(t,y) | \quad -1 \le t \le 1, \quad -1 \le y \le 1\}$

10. Find the $\|A\|_\infty$ of the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

(a) 4
(b) 12
(c) 1
(d) 2
(e) 11

11. Use the Gaussian Elimination Algorithm (exact arithmetic, *no rounding*) to solve

$$\begin{cases} x_1 - x_2 + 3x_3 = 2, \\ 3x_1 - 3x_2 + x_3 = -1, \\ x_1 + x_2 \qquad = 3. \end{cases}$$

Then, $x_1 + x_2 + x_3 =$.

(a) 3.5
(b) 3.875
(c) 3.625
(d) 4.000
(e) 3.750

12. If

$$A = \begin{bmatrix} a & 1 & 0 \\ b & 2 & 3 \\ c & 0 & 1 \end{bmatrix}, \qquad \text{find the permutation matrix } P \text{ such that} \quad PA = \begin{bmatrix} c & 0 & 1 \\ b & 2 & 3 \\ a & 1 & 0 \end{bmatrix}.$$

The matrix $P$ is

(a) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

13. Consider the system

$$\begin{cases} 4x_1 + x_2 - x_3 = 5, \\ -x_1 + 3x_2 + x_3 = -4, \\ 2x_1 + 2x_2 + 5x_3 = 1. \end{cases}$$

The first iterate of the Gauss-Seidel method in solving the system with initial guess $\mathbf{x}^{(0)} = (0, 0, 0)^t$, is $\mathbf{x}^{(1)} =$

(a) $\left(1.4000000000,\ -0.9000000000,\ 0.0000000000\right)^t$

(b) $\left(1.2500000000,\ -1.3333333333,\ 0.2000000000\right)^t$

(c) $\left(0.9000000000,\ 0.7000000000,\ 0.6000000000\right)^t$

(d) $\left(1.4958333333,\ -0.8569444444,\ -0.0555555556\right)^t$

(e) $\left(1.250000000,\ -0.9166666667,\ 0.0666666667\right)^t$

14. Consider the initial value problem

$$y' = y - t^2 + 1, \qquad 0 \le t \le 2, \qquad y(0) = 0.5, \qquad h = 0.1.$$

If the fourth–order Runge–Kutta method is used to approximate the solution, then $y(0.2) \approx$

(a) 1.2056345

(b) 0.6574145

(c) 1.0150701

(d) 0.8292983

(e) 1.2140869

King Fahd University of Petroleum and Minerals
Department of Mathematics

**CODE07** **CODE07**

# Math 371
# Major Exam II
# 251
# November 9, 2025
# Net Time Allowed: 90 Minutes

| **Name** | | | |
|---|---|---|---|
| **ID** | | **Sec** | |

**Check that this exam has 14 questions.**

## Important Instructions:

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8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. Use the LU Factorization Algorithm to factor

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \\ -1 & 3 & 2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}}_{U}.$$

Compute $\ell_{32} + u_{33}$.

(a) 3/2
(b) 1
(c) 1/2
(d) 0
(e) 2

2. Use the Gaussian Elimination Algorithm (exact arithmetic, *no rounding*) to solve

$$\begin{cases} x_1 - x_2 + 3x_3 = 2, \\ 3x_1 - 3x_2 + x_3 = -1, \\ x_1 + x_2 \qquad = 3. \end{cases}$$

Then, $x_1 + x_2 + x_3 =$.

(a) 3.875
(b) 3.750
(c) 3.5
(d) 3.625
(e) 4.000

3. Consider the initial value problem

$$y' = y - t^2 + 1, \qquad 0 \le t \le 2, \qquad y(0) = 0.5, \qquad h = 0.1.$$

If the fourth–order Runge–Kutta method is used to approximate the solution, then $y(0.2) \approx$

(a) 0.6574145
(b) 1.0150701
(c) 1.2140869
(d) 1.2056345
(e) 0.8292983

4. Which one of the following functions does not satisfy a Lipschitz condition on the domain $D$

(a) $f(t, y) = (t^2 + 1)y^3$ on $D = \{(t, y)| \quad -1 \le t \le 1, \quad -1 \le y \le 1\}$
(b) $f(t, y) = \dfrac{y}{1 + t^2}$ on $D = \{(t, y)| \quad 0 \le t \le 3, \quad -2 \le y \le 2\}$
(c) $f(t, y) = y \ln y + t$ on $D = \{(t, y)| \quad 0 \le t \le 1, \quad 0 \le y \le 1\}$
(d) $f(t, y) = e^t \sin y$ on $D = \{(t, y)| \quad 0 \le t \le 2, \quad -\pi \le y \le \pi\}$
(e) $f(t, y) = ye^{y^2} + t^3$ on $D = \{(t, y)| \quad -1 \le t \le 1, \quad -1 \le y \le 1\}$

5. Find the $\|A\|_\infty$ of the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

(a) 1
(b) 12
(c) 2
(d) 4
(e) 11

6. Let

$$\mathbf{x} = \begin{pmatrix} 3 \\ -4 \\ 0 \\ \frac{3}{2} \end{pmatrix} \in \mathbb{R}^4.$$

Which statement about the norms of $\mathbf{x}$ is correct?

(a) $\|\mathbf{x}\|_2 + \|\mathbf{x}\|_\infty < 9$
(b) $\|\mathbf{x}\|_2 > \|\mathbf{x}\|_\infty$
(c) $\|\mathbf{x}\|_2 - \|\mathbf{x}\|_\infty > 9$
(d) $\|\mathbf{x}\|_2 < \|\mathbf{x}\|_\infty$
(e) $\|\mathbf{x}\|_2 = \|\mathbf{x}\|_\infty$

7. If

$$A = \begin{bmatrix} a & 1 & 0 \\ b & 2 & 3 \\ c & 0 & 1 \end{bmatrix}, \qquad \text{find the permutation matrix } P \text{ such that} \quad PA = \begin{bmatrix} c & 0 & 1 \\ b & 2 & 3 \\ a & 1 & 0 \end{bmatrix}.$$

The matrix $P$ is

(a) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(e) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

8. Consider the initial value problem

$$y' = \tfrac{1}{2}y + t, \qquad 0 \le t \le 1, \qquad h = 0.2, \qquad y(0) = 1,$$

with exact solution $y(t) = 5e^{t/2} - 2t - 4$. If Euler's method is used to approximate the solution, then the least bound for $|y(1) - w_5|$ is

(a) 0.506500

(b) 0.714785

(c) 0.267390

(d) 0.801250

(e) 0.419500

9. Consider the linear system

$$\begin{aligned} -6x_1 - 12x_2 - x_3 &= -9, \\ 2x_1 + x_2 - x_3 &= 8, \\ 5x_1 + 12x_2 + x_3 &= 7. \end{aligned}$$

Let $R_i^{(k)}$ denote the $i$-th row of the augmented matrix at stage $k$ of Gaussian elimination and let $R_i^{(k)} \leftrightarrow R_j^{(k)}$ denote an interchange of the $i$-th and $j$-th rows. The row interchanges required to solve the system by *partial pivoting* are

(a) Stage 1: $R_1^{(1)} \leftrightarrow R_3^{(1)}$, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$
(b) Stage 1: $R_1^{(1)} \leftrightarrow R_2^{(1)}$, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$
(c) Stage 1: $R_1^{(1)} \leftrightarrow R_3^{(1)}$, Stage 2: No row interchanges
(d) Stage 1: No row interchanges, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$
(e) No row interchanges required in stages 1 and 2

10. For the linear system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix},$$

an approximate solution is $\tilde{\mathbf{x}} = \begin{bmatrix} -0.2 \\ -7.5 \\ 5.4 \end{bmatrix}$. Compute $\|A\tilde{\mathbf{x}} - \mathbf{b}\|_\infty$.

(a) 0.3
(b) 0.1
(c) 0.2
(d) 0.5
(e) 0.4

11. Consider the initial value problem

$$y' = te^{3t} - 2y, \qquad 0 \le t \le 1, \qquad y(0) = 0, \qquad h = 0.5,$$

with exact solution

$$y(t) = \frac{1}{5}te^{3t} - \frac{1}{25}e^{3t} + \frac{1}{25}e^{-2t}.$$

If the midpoint method is used to approximate the solution, then the absolute error at $t = 1$ is

(a) 0.0972
(b) 0.0312
(c) 0.0642
(d) 0.0792
(e) 0.0891

12. Let

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

Find the spectral radius $\rho(A)$.

(a) 1
(b) 2
(c) 3
(d) 5
(e) 4

13. Consider the system

$$\begin{cases} 4x_1 + x_2 - x_3 = 5, \\ -x_1 + 3x_2 + x_3 = -4, \\ 2x_1 + 2x_2 + 5x_3 = 1. \end{cases}$$

The first iterate of the Gauss-Seidel method in solving the system with initial guess $\mathbf{x}^{(0)} = (0, 0, 0)^t$, is $\mathbf{x}^{(1)} =$

(a) $\left(1.4958333333,\ -0.8569444444,\ -0.0555555556\right)^t$
(b) $\left(0.9000000000,\ 0.7000000000,\ 0.6000000000\right)^t$
(c) $\left(1.4000000000,\ -0.9000000000,\ 0.0000000000\right)^t$
(d) $\left(1.250000000,\ -0.9166666667,\ 0.0666666667\right)^t$
(e) $\left(1.2500000000,\ -1.3333333333,\ 0.2000000000\right)^t$

14. Let $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ be the first and second iterates, respectively, of the Jacobi method with $\mathbf{x}^{(0)} = (1, 0, -1)^\top$ for the system

$$\begin{cases} 10x_1 - x_2 = 9, \\ -x_1 + 10x_2 - 2x_3 = 7, \\ -2x_2 + 10x_3 = 6. \end{cases}$$

Then $\left\|\mathbf{x}^{(2)} - \mathbf{x}^{(1)}\right\|_\infty =$

(a) 0.21
(b) 0.06
(c) 0.14
(d) 0.12
(e) 0.31

King Fahd University of Petroleum and Minerals
Department of Mathematics

| CODE08 | | CODE08 |
|---|---|---|

**Math 371**
**Major Exam II**
**251**
**November 9, 2025**
**Net Time Allowed: 90 Minutes**

| **Name** | | | |
|---|---|---|---|
| **ID** | | **Sec** | |

**Check that this exam has 14 questions.**

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1. Use the LU Factorization Algorithm to factor

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \\ -1 & 3 & 2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}}_{U}.$$

Compute $\ell_{32} + u_{33}$.

(a) 1
(b) 2
(c) 0
(d) 1/2
(e) 3/2

2. Consider the initial value problem

$$y' = y - t^2 + 1, \qquad 0 \le t \le 2, \qquad y(0) = 0.5, \qquad h = 0.1.$$

If the fourth–order Runge–Kutta method is used to approximate the solution, then $y(0.2) \approx$

(a) 0.8292983
(b) 1.2140869
(c) 1.0150701
(d) 1.2056345
(e) 0.6574145

3. Consider the initial value problem

$$y' = te^{3t} - 2y, \qquad 0 \le t \le 1, \qquad y(0) = 0, \qquad h = 0.5,$$

with exact solution

$$y(t) = \frac{1}{5}te^{3t} - \frac{1}{25}e^{3t} + \frac{1}{25}e^{-2t}.$$

If the midpoint method is used to approximate the solution, then the absolute error at $t = 1$ is

(a) 0.0642
(b) 0.0972
(c) 0.0792
(d) 0.0891
(e) 0.0312

4. Which one of the following functions does not satisfy a Lipschitz condition on the domain $D$

(a) $f(t, y) = y \ln y + t$ on $D = \{(t, y)| \quad 0 \le t \le 1, \quad 0 \le y \le 1\}$
(b) $f(t, y) = \dfrac{y}{1 + t^2}$ on $D = \{(t, y)| \quad 0 \le t \le 3, \quad -2 \le y \le 2\}$
(c) $f(t, y) = (t^2 + 1)y^3$ on $D = \{(t, y)| \quad -1 \le t \le 1, \quad -1 \le y \le 1\}$
(d) $f(t, y) = e^t \sin y$ on $D = \{(t, y)| \quad 0 \le t \le 2, \quad -\pi \le y \le \pi\}$
(e) $f(t, y) = ye^{y^2} + t^3$ on $D = \{(t, y)| \quad -1 \le t \le 1, \quad -1 \le y \le 1\}$

5. Find the $\|A\|_\infty$ of the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

(a) 12
(b) 1
(c) 11
(d) 4
(e) 2

6. Consider the linear system

$$\begin{aligned} -6x_1 - 12x_2 - x_3 &= -9, \\ 2x_1 + x_2 - x_3 &= 8, \\ 5x_1 + 12x_2 + x_3 &= 7. \end{aligned}$$

Let $R_i^{(k)}$ denote the $i$-th row of the augmented matrix at stage $k$ of Gaussian elimination and let $R_i^{(k)} \leftrightarrow R_j^{(k)}$ denote an interchange of the $i$-th and $j$-th rows. The row interchanges required to solve the system by *partial pivoting* are

(a) Stage 1: $R_1^{(1)} \leftrightarrow R_3^{(1)}$, Stage 2: No row interchanges
(b) Stage 1: $R_1^{(1)} \leftrightarrow R_3^{(1)}$, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$
(c) No row interchanges required in stages 1 and 2
(d) Stage 1: No row interchanges, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$
(e) Stage 1: $R_1^{(1)} \leftrightarrow R_2^{(1)}$, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$

7. Let $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ be the first and second iterates, respectively, of the Jacobi method with $\mathbf{x}^{(0)} = (1, 0, -1)^\mathsf{T}$ for the system

$$\begin{cases} 10x_1 - x_2 = 9, \\ -x_1 + 10x_2 - 2x_3 = 7, \\ -2x_2 + 10x_3 = 6. \end{cases}$$

Then $\left\|\mathbf{x}^{(2)} - \mathbf{x}^{(1)}\right\|_\infty =$

(a) 0.14
(b) 0.12
(c) 0.31
(d) 0.21
(e) 0.06

8. If

$$A = \begin{bmatrix} a & 1 & 0 \\ b & 2 & 3 \\ c & 0 & 1 \end{bmatrix}, \qquad \text{find the permutation matrix } P \text{ such that} \quad PA = \begin{bmatrix} c & 0 & 1 \\ b & 2 & 3 \\ a & 1 & 0 \end{bmatrix}.$$

The matrix $P$ is

(a) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

9. Let

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

Find the spectral radius $\rho(A)$.

(a) 3
(b) 1
(c) 5
(d) 4
(e) 2

10. Consider the initial value problem

$$y' = \tfrac{1}{2}y + t, \qquad 0 \le t \le 1, \qquad h = 0.2, \qquad y(0) = 1,$$

with exact solution $y(t) = 5e^{t/2} - 2t - 4$. If Euler's method is used to approximate the solution, then the least bound for $|y(1) - w_5|$ is

(a) 0.267390
(b) 0.714785
(c) 0.801250
(d) 0.506500
(e) 0.419500

11. Use the Gaussian Elimination Algorithm (exact arithmetic, *no rounding*) to solve

$$\begin{cases} x_1 - x_2 + 3x_3 = 2, \\ 3x_1 - 3x_2 + x_3 = -1, \\ x_1 + x_2 \qquad = 3. \end{cases}$$

Then, $x_1 + x_2 + x_3 =$.

(a) 4.000
(b) 3.750
(c) 3.5
(d) 3.875
(e) 3.625

12. For the linear system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix},$$

an approximate solution is $\tilde{\mathbf{x}} = \begin{bmatrix} -0.2 \\ -7.5 \\ 5.4 \end{bmatrix}$. Compute $\|A\tilde{\mathbf{x}} - \mathbf{b}\|_\infty$.

(a) 0.5
(b) 0.2
(c) 0.1
(d) 0.4
(e) 0.3

13. Let

$$\mathbf{x} = \begin{pmatrix} 3 \\ -4 \\ 0 \\ \frac{3}{2} \end{pmatrix} \in \mathbb{R}^4.$$

Which statement about the norms of $\mathbf{x}$ is correct?

(a) $\|\mathbf{x}\|_2 - \|\mathbf{x}\|_\infty > 9$

(b) $\|\mathbf{x}\|_2 > \|\mathbf{x}\|_\infty$

(c) $\|\mathbf{x}\|_2 + \|\mathbf{x}\|_\infty < 9$

(d) $\|\mathbf{x}\|_2 = \|\mathbf{x}\|_\infty$

(e) $\|\mathbf{x}\|_2 < \|\mathbf{x}\|_\infty$

14. Consider the system

$$\begin{cases} 4x_1 + x_2 - x_3 = 5, \\ -x_1 + 3x_2 + x_3 = -4, \\ 2x_1 + 2x_2 + 5x_3 = 1. \end{cases}$$

The first iterate of the Gauss-Seidel method in solving the system with initial guess $\mathbf{x}^{(0)} = (0, 0, 0)^{\mathrm{t}}$, is $\mathbf{x}^{(1)} =$

(a) $\big(1.2500000000,\ -1.3333333333,\ 0.2000000000\big)^{\mathrm{t}}$

(b) $\big(1.4958333333,\ -0.8569444444,\ -0.0555555556\big)^{\mathrm{t}}$

(c) $\big(1.4000000000,\ -0.9000000000,\ 0.0000000000\big)^{\mathrm{t}}$

(d) $\big(0.9000000000,\ 0.7000000000,\ 0.6000000000\big)^{\mathrm{t}}$

(e) $\big(1.250000000,\ -0.9166666667,\ 0.0666666667\big)^{\mathrm{t}}$

**Answer KEY**

| Q | MASTER | CODE01 | CODE02 | CODE03 | CODE04 | CODE05 | CODE06 | CODE07 |
|---|---|---|---|---|---|---|---|---|
| 1 | A | E 11 | B 5 | D 14 | B 12 | A 2 | D 3 | B 9 |
| 2 | A | E 3 | A 4 | E 7 | D 9 | A 6 | E 11 | A 5 |
| 3 | A | B 6 | E 1 | D 10 | B 6 | C 8 | D 6 | E 4 |
| 4 | A | A 1 | D 2 | D 5 | D 1 | C 5 | E 9 | C 1 |
| 5 | A | D 12 | D 13 | D 4 | E 10 | A 3 | D 12 | D 10 |
| 6 | A | C 14 | E 12 | A 2 | D 8 | D 13 | D 2 | B 8 |
| 7 | A | D 2 | A 7 | E 6 | A 13 | D 14 | B 13 | B 7 |
| 8 | A | C 10 | B 14 | B 13 | E 11 | B 1 | D 8 | C 2 |
| 9 | A | C 13 | A 10 | D 8 | E 5 | E 9 | D 1 | E 6 |
| 10 | A | C 8 | A 3 | E 12 | C 4 | C 10 | A 10 | A 11 |
| 11 | A | D 5 | B 8 | B 3 | A 14 | D 11 | B 5 | E 3 |
| 12 | A | D 7 | D 6 | D 1 | C 7 | B 12 | B 7 | C 12 |
| 13 | A | A 4 | B 11 | A 9 | A 3 | D 7 | E 14 | D 14 |
| 14 | A | E 9 | B 9 | E 11 | E 2 | D 4 | D 4 | E 13 |

 **Answer KEY**

| Q | MASTER | CODE08 |
|---|---|---|
| 1 | A | A 9 |
| 2 | A | A 4 |
| 3 | A | D 3 |
| 4 | A | A 1 |
| 5 | A | D 10 |
| 6 | A | C 6 |
| 7 | A | C 13 |
| 8 | A | E 7 |
| 9 | A | A 12 |
| 10 | A | A 2 |
| 11 | A | D 5 |
| 12 | A | E 11 |
| 13 | A | B 8 |
| 14 | A | E 14 |

# Answer Counts

| V | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | 2 | 1 | 4 | 4 | 3 |
| 2 | 4 | 5 | 0 | 3 | 2 |
| 3 | 2 | 2 | 0 | 6 | 4 |
| 4 | 3 | 2 | 2 | 3 | 4 |
| 5 | 3 | 2 | 3 | 5 | 1 |
| 6 | 1 | 3 | 0 | 7 | 3 |
| 7 | 2 | 3 | 3 | 2 | 4 |
| 8 | 5 | 1 | 2 | 3 | 3 |