King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 371
Final Exam
251
December 15, 2025

EXAM COVER

Number of versions: 4
Number of questions: 20

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Department of Mathematics

Math 371

Final Exam

251

December 15, 2025

Net Time Allowed: 120 Minutes

MASTER VERSION

1. Let $P_2(x)$ be the second Taylor polynomial for $f(x) = \ln(x^2 + 3)$ about $x_0 = 0$. By Taylor's theorem, the smallest upper bound you can justify for $|f(1.5) - P_2(1.5)|$ is

   (a) 3.15×10^{-1} ____________________ (correct)
   (b) 2.50×10^{-1}
   (c) 3.50×10^{-1}
   (d) 1.00×10^{-1}
   (e) 5.00×10^{-2}

2. Consider $f(x) = 5x - 12\pi + 7e$. Assume three-digit rounding floating-point arithmetic is used, with rounding after every basic operation (each addition, subtraction, multiplication, or division). Use the three-digit approximations $\pi \approx 3.14$ and $e \approx 2.72$, and evaluate $f(x)$ at $x = 17/61$. In this arithmetic, $f(17/61)$ is approximately

   (a) -17.3 ____________________ (correct)
   (b) -17.1
   (c) -17.2
   (d) -17.4
   (e) -17.5

3. Apply Newton's Method to approximate the x-value of the intersection point of the graphs of

$$f(x) = \ln(1 + x^2) \quad \text{and} \quad g(x) = 2 - e^{-x}.$$

Use $p_0 = 0.5$. Continue the iterations until $|p_N - p_{N-1}| < 0.4$. Then p_N is

(a) 2.3971 ________ (correct)
(b) 1.3321
(c) 4.9937
(d) 1.6564
(e) 2.1874

4. If the quadratic Lagrange interpolating polynomial through the points $(0, 0)$, $(0.6, 0.6841)$ and $(1.2, 1.3133)$ is used to approximate $f(x)$, then $f(0.8) \approx$

(a) 0.8999 ________ (correct)
(b) 0.9121
(c) 0.8734
(d) 0.8840
(e) 0.9275

5. Given the following data

$$f(1.4) = 14.1, \qquad f(1.5) = 15.6, \qquad f(1.6) = 18.5,$$

use the three-point midpoint and endpoint formulas to approximate $f'(1.5)+f'(1.6)$. Then

$$f'(1.5) + f'(1.6) \approx$$

(a) 58 ________________ (correct)

(b) 56

(c) 60

(d) 54

(e) 62

6. Using the Composite Simpson's rule with $n = 4$ subintervals to approximate

$$\int_0^2 x^2 \cos x \, dx,$$

we obtain

(a) 0.1550 ________________ (correct)

(b) 0.1415

(c) 0.1632

(d) 0.1320

(e) 0.1789

7. Using Gaussian elimination with partial pivoting *and three-digit chopping arithmetic* to solve

$$\begin{cases} ex + \pi y = 1, \\ \pi x + ey = 0, \end{cases}$$

let $\mathbf{u} = (x, y)^T$ be the solution vector. Then $\|\mathbf{u}\|_\infty \approx$

(a) 1.23 ______________________________ (correct)

(b) 2.43

(c) 1.06

(d) 2.05

(e) 2.76

8. Consider the initial value problem

$$y' = y, \qquad 0 \le t \le 0.5, \qquad y(0) = 1,$$

whose exact solution is $y(t) = e^t$.

Apply Euler's method with step size $h = 0.25$ to obtain the approximation w_2 to $y(0.5)$ after two steps. Using the global error bound for Euler's method, an upper bound for $|y(0.5) - w_2|$ is

(a) $\dfrac{e - \sqrt{e}}{8}$ ______________________________ (correct)

(b) $\dfrac{e - 1}{8}$

(c) $\dfrac{\sqrt{e} - 1}{8}$

(d) $\dfrac{e - \sqrt{e}}{4}$

(e) $\dfrac{e - \sqrt{e}}{16}$

9. Consider the initial value problem

$$y' = \cos(2t) + \sin(3t), \qquad 0 \le t \le 1, \qquad y(0) = 1,$$

whose exact solution is

$$y(t) = \frac{1}{2}\sin(2t) - \frac{1}{3}\cos(3t) + \frac{4}{3}.$$

Using the classical fourth-order Runge–Kutta method with step size $h = 0.5$ (two steps) to compute the approximation w_2 to $y(1)$, the actual error

$$|y(1) - w_2|$$

is approximately

(a) 1.41×10^{-3} ______ (correct)

(b) 1.00×10^{-2}

(c) 5.00×10^{-3}

(d) 1.00×10^{-4}

(e) 5.00×10^{-4}

10. Let

$$A = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix},$$

with $a > b > c > 0$. If $(\rho(A) + \rho(A^T))\,\|A\|_\infty = 2k$, then $k =$

(a) $a^2 + ab$ ______ (correct)

(b) $a + b$

(c) $2a + 2ab$

(d) $a^2 + 2ab$

(e) $a + ab^2$

11. Let $x^{(0)} = (0, 0, 0)$. Consider applying the Gauss–Seidel method to the system

$$\begin{cases} 4x_1 - x_2 + x_3 = 7, \\ -x_1 + 3x_2 - 2x_3 = -4, \\ 2x_1 + x_2 + 5x_3 = 1. \end{cases}$$

Let $x^{(1)} = \left(x_1^{(1)}, x_2^{(1)}, x_3^{(1)}\right)$ and $x^{(2)} = \left(x_1^{(2)}, x_2^{(2)}, x_3^{(2)}\right)$ be the first two Gauss–Seidel iterates. Then $x_3^{(1)} + x_2^{(2)} \approx$

(a) -1.3667 ______________________________ (correct)

(b) -1.2833

(c) -1.2567

(d) -1.1833

(e) -1.0167

12. Let $P(x) = \frac{1}{5}x + b$ be the least squares polynomial of degree one for the data in the table

x_i	-1	0	1	α
y_i	2	-3	5	0

where $\alpha > 0$. Then $P(0) =$

(a) 0.9000 ______________________________ (correct)

(b) 0.7500

(c) 1.0500

(d) 0.5000

(e) 1.2000

13. Let $\{(1,2,-1)^t,\ (2,1,3)^t\}$ be a set of two linearly independent vectors in $\mathbb{R}^3$. By the Gram–Schmidt process the orthogonal set is $\{(1,2,-1)^t,\ V\}$. Then $V =$

(a) $(11,4,19)^t$ ____(correct)

(b) $(11,4,-19)^t$

(c) $(1,0,3)^t$

(d) $(5,2,7)^t$

(e) $(4,11,19)^t$

14. Let

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & a \end{pmatrix}, \qquad a > 0,$$

be a 3×2 matrix. Suppose that 2 is a singular value of A. Then $a =$

(a) $\sqrt{2}$ ____(correct)

(b) 1

(c) 2

(d) $\dfrac{1}{\sqrt{2}}$

(e) 4

15. Let A be a real $m \times n$ matrix with $m \geq n$, and let

$$A = USV^t$$

be a singular value decomposition of A. Which of the following statements is *not always true*?

(a) If A is square and invertible, then all singular values of A are equal to 1. (correct)
(b) The rank of A equals the number of nonzero diagonal entries of S.
(c) The columns of U corresponding to nonzero singular values form an orthonormal basis for the column space of A.
(d) U and V care always invertible.
(e) The matrix S has nonnegative entries on its main diagonal and zeros elsewhere.

16. Let $\mathbf{x} = (x_1, x_2)^t$ and let $\mathbf{x}^{(1)}$ be the first iterate of Newton's method applied to the nonlinear system

$$\begin{cases} x_1^2 + x_2^2 - 5 = 0, \\ x_1^2 - x_2 - 1 = 0, \end{cases}$$

with initial guess $\mathbf{x}^{(0)} = (1, 1)^t$. Let $\| \cdot \|_2$ denote the Euclidean norm on $\mathbb{R}^2$. Then

$$\left\|\mathbf{x}^{(1)}\right\|_2^2 \approx$$

(a) 6.14 ____(correct)
(b) 5.75
(c) 4.50
(d) 7.00
(e) 3.25

17. Let $\mathbf{x} = (x_1, x_2)^t$ and let $\mathbf{x}^{(1)}$ be the first iterate of the steepest descent method in solving the nonlinear system

$$\begin{cases} x_1^2 + 4x_2^2 = 5, \\ x_1 x_2 = 1, \end{cases}$$

Take the initial guess $\mathbf{x}^{(0)} = (1, \frac{1}{2})^t$ and step size $\alpha = 0.02$. Then $\mathbf{x}^{(1)}$ is approximately

(a) $(1.1250, 0.7500)^t$ ____(correct)
(b) $(1.1250, 0.6500)^t$
(c) $(1.0500, 0.7500)^t$
(d) $(0.8750, 0.7500)^t$
(e) $(1.1250, 0.8500)^t$

18. Given the boundary-value problem

$$y'' = 9y, \qquad 0 \le x \le 1, \qquad y(0) = 0,\ y(1) = 0.$$

Using the linear finite difference method with step size $h = \dfrac{1}{3}$ to obtain a system

$$A\mathbf{w} = \mathbf{b}$$

for the interior approximations $\mathbf{w} = (w_1, w_2)^t$ (at $x_1 = \frac{1}{3}$ and $x_2 = \frac{2}{3}$), the sum of the diagonal entries of A is

(a) -6 ____(correct)
(b) -4
(c) -2
(d) 0
(e) 2

19. Given the boundary-value problem

$$y'' = 2y' - y + x, \qquad 0 \le x \le 1, \qquad y(0) = 3,\ y(1) = 4.$$

Using the Linear Finite-Difference method with step size $h = 0.5$, $y(0.5) \approx$

(a) 3.64 ____(correct)

(b) 3.50

(c) 3.75

(d) 3.25

(e) 3.90

20. If $A = LU$, where

$$L = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{pmatrix}, \quad U \text{ is upper triangular, and} \quad A = \begin{pmatrix} 2 & 1 & -1 \\ 4 & 3 & 1 \\ 6 & 5 & 2 \end{pmatrix},$$

then $a + b + 2c =$

(a) 9 ____(correct)

(b) 6

(c) 5

(d) 3

(e) 1

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Department of Mathematics

CODE01 **CODE01**

Math 371
Final Exam
251
December 15, 2025
Net Time Allowed: 120 Minutes

Name	

ID		**Sec**	

Check that this exam has 20 questions.

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2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
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6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
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8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. If $A = LU$, where

$$L = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{pmatrix}, \quad U \text{ is upper triangular, and} \quad A = \begin{pmatrix} 2 & 1 & -1 \\ 4 & 3 & 1 \\ 6 & 5 & 2 \end{pmatrix},$$

then $a + b + 2c =$

(a) 5
(b) 3
(c) 1
(d) 6
(e) 9

2. If the quadratic Lagrange interpolating polynomial through the points $(0, 0)$, $(0.6, 0.6841)$ and $(1.2, 1.3133)$ is used to approximate $f(x)$, then $f(0.8) \approx$

(a) 0.8734
(b) 0.8999
(c) 0.9275
(d) 0.9121
(e) 0.8840

3. Let $\mathbf{x} = (x_1, x_2)^t$ and let $\mathbf{x}^{(1)}$ be the first iterate of the steepest descent method in solving the nonlinear system

$$\begin{cases} x_1^2 + 4x_2^2 = 5, \\ x_1 x_2 = 1, \end{cases}$$

Take the initial guess $\mathbf{x}^{(0)} = (1, \frac{1}{2})^t$ and step size $\alpha = 0.02$. Then $\mathbf{x}^{(1)}$ is approximately

(a) $(0.8750, 0.7500)^t$
(b) $(1.1250, 0.7500)^t$
(c) $(1.0500, 0.7500)^t$
(d) $(1.1250, 0.6500)^t$
(e) $(1.1250, 0.8500)^t$

4. Given the following data

$$f(1.4) = 14.1, \qquad f(1.5) = 15.6, \qquad f(1.6) = 18.5,$$

use the three-point midpoint and endpoint formulas to approximate $f'(1.5)+f'(1.6)$. Then

$$f'(1.5) + f'(1.6) \approx$$

(a) 54
(b) 56
(c) 62
(d) 58
(e) 60

5. Given the boundary-value problem

$$y'' = 9y, \qquad 0 \le x \le 1, \qquad y(0) = 0,\ y(1) = 0.$$

Using the linear finite difference method with step size $h = \dfrac{1}{3}$ to obtain a system

$$A\mathbf{w} = \mathbf{b}$$

for the interior approximations $\mathbf{w} = (w_1, w_2)^t$ (at $x_1 = \frac{1}{3}$ and $x_2 = \frac{2}{3}$), the sum of the diagonal entries of A is

(a) -2
(b) -4
(c) 2
(d) 0
(e) -6

6. Consider the initial value problem

$$y' = y, \qquad 0 \le t \le 0.5, \qquad y(0) = 1,$$

whose exact solution is $y(t) = e^t$.

Apply Euler's method with step size $h = 0.25$ to obtain the approximation w_2 to $y(0.5)$ after two steps. Using the global error bound for Euler's method, an upper bound for $|y(0.5) - w_2|$ is

(a) $\dfrac{\sqrt{e} - 1}{8}$
(b) $\dfrac{e - \sqrt{e}}{4}$
(c) $\dfrac{e - 1}{8}$
(d) $\dfrac{e - \sqrt{e}}{8}$
(e) $\dfrac{e - \sqrt{e}}{16}$

7. Let A be a real $m \times n$ matrix with $m \geq n$, and let

$$A = USV^t$$

be a singular value decomposition of A. Which of the following statements is *not always true*?

(a) If A is square and invertible, then all singular values of A are equal to 1.
(b) The rank of A equals the number of nonzero diagonal entries of S.
(c) The columns of U corresponding to nonzero singular values form an orthonormal basis for the column space of A.
(d) U and V care always invertible.
(e) The matrix S has nonnegative entries on its main diagonal and zeros elsewhere.

8. Let

$$A = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix},$$

with $a > b > c > 0$. If $(\rho(A) + \rho(A^T)) \|A\|_\infty = 2k$, then $k =$

(a) $a^2 + ab$
(b) $a + b$
(c) $2a + 2ab$
(d) $a^2 + 2ab$
(e) $a + ab^2$

9. Let $P(x) = \frac{1}{5}x + b$ be the least squares polynomial of degree one for the data in the table

x_i	-1	0	1	α
y_i	2	-3	5	0

where $\alpha > 0$. Then $P(0) =$

(a) 1.2000
(b) 0.7500
(c) 0.9000
(d) 1.0500
(e) 0.5000

10. Let $P_2(x)$ be the second Taylor polynomial for $f(x) = \ln(x^2 + 3)$ about $x_0 = 0$. By Taylor's theorem, the smallest upper bound you can justify for $|f(1.5) - P_2(1.5)|$ is

(a) 1.00×10^{-1}
(b) 3.50×10^{-1}
(c) 3.15×10^{-1}
(d) 2.50×10^{-1}
(e) 5.00×10^{-2}

11. Let $\{(1,2,-1)^t,\ (2,1,3)^t\}$ be a set of two linearly independent vectors in $\mathbb{R}^3$. By the Gram–Schmidt process the orthogonal set is $\{(1,2,-1)^t,\ V\}$. Then $V =$

    (a) $(4,11,19)^t$
    (b) $(11,4,19)^t$
    (c) $(1,0,3)^t$
    (d) $(11,4,-19)^t$
    (e) $(5,2,7)^t$

12. Let $x^{(0)} = (0,0,0)$. Consider applying the Gauss–Seidel method to the system

$$\begin{cases} 4x_1 - x_2 + x_3 = 7, \\ -x_1 + 3x_2 - 2x_3 = -4, \\ 2x_1 + x_2 + 5x_3 = 1. \end{cases}$$

Let $x^{(1)} = \left(x_1^{(1)}, x_2^{(1)}, x_3^{(1)}\right)$ and $x^{(2)} = \left(x_1^{(2)}, x_2^{(2)}, x_3^{(2)}\right)$ be the first two Gauss–Seidel iterates. Then $x_3^{(1)} + x_2^{(2)} \approx$

    (a) -1.2567
    (b) -1.1833
    (c) -1.3667
    (d) -1.2833
    (e) -1.0167

13. Using the Composite Simpson's rule with $n = 4$ subintervals to approximate

$$\int_0^2 x^2 \cos x \, dx,$$

we obtain

(a) 0.1415
(b) 0.1550
(c) 0.1789
(d) 0.1320
(e) 0.1632

14. Using Gaussian elimination with partial pivoting *and three-digit chopping arithmetic* to solve

$$\begin{cases} ex + \pi y = 1, \\ \pi x + ey = 0, \end{cases}$$

let $\mathbf{u} = (x, y)^T$ be the solution vector. Then $\|\mathbf{u}\|_\infty \approx$

(a) 1.06
(b) 2.05
(c) 1.23
(d) 2.76
(e) 2.43

15. Consider $f(x) = 5x - 12\pi + 7e$. Assume three-digit rounding floating-point arithmetic is used, with rounding after every basic operation (each addition, subtraction, multiplication, or division). Use the three-digit approximations $\pi \approx 3.14$ and $e \approx 2.72$, and evaluate $f(x)$ at $x = 17/61$. In this arithmetic, $f(17/61)$ is approximately

(a) -17.5
(b) -17.2
(c) -17.4
(d) -17.1
(e) -17.3

16. Let

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & a \end{pmatrix}, \qquad a > 0,$$

be a 3×2 matrix. Suppose that 2 is a singular value of A. Then $a =$

(a) 2
(b) $\sqrt{2}$
(c) 1
(d) 4
(e) $\dfrac{1}{\sqrt{2}}$

17. Let $\mathbf{x} = (x_1, x_2)^t$ and let $\mathbf{x}^{(1)}$ be the first iterate of Newton's method applied to the nonlinear system

$$\begin{cases} x_1^2 + x_2^2 - 5 = 0, \\ x_1^2 - x_2 - 1 = 0, \end{cases}$$

with initial guess $\mathbf{x}^{(0)} = (1, 1)^t$. Let $\| \cdot \|_2$ denote the Euclidean norm on $\mathbb{R}^2$. Then

$$\left\|\mathbf{x}^{(1)}\right\|_2^2 \approx$$

(a) 6.14
(b) 3.25
(c) 5.75
(d) 7.00
(e) 4.50

18. Consider the initial value problem

$$y' = \cos(2t) + \sin(3t), \qquad 0 \leq t \leq 1, \qquad y(0) = 1,$$

whose exact solution is

$$y(t) = \frac{1}{2}\sin(2t) - \frac{1}{3}\cos(3t) + \frac{4}{3}.$$

Using the classical fourth-order Runge–Kutta method with step size $h = 0.5$ (two steps) to compute the approximation w_2 to $y(1)$, the actual error

$$|y(1) - w_2|$$

is approximately

(a) 1.00×10^{-4}
(b) 5.00×10^{-3}
(c) 5.00×10^{-4}
(d) 1.00×10^{-2}
(e) 1.41×10^{-3}

19. Given the boundary-value problem

$$y'' = 2y' - y + x, \qquad 0 \le x \le 1, \qquad y(0) = 3,\ y(1) = 4.$$

Using the Linear Finite-Difference method with step size $h = 0.5$, $y(0.5) \approx$

(a) 3.75
(b) 3.25
(c) 3.64
(d) 3.50
(e) 3.90

20. Apply Newton's Method to approximate the x-value of the intersection point of the graphs of

$$f(x) = \ln(1 + x^2) \quad \text{and} \quad g(x) = 2 - e^{-x}.$$

Use $p_0 = 0.5$. Continue the iterations until $|p_N - p_{N-1}| < 0.4$. Then p_N is

(a) 1.6564
(b) 4.9937
(c) 2.1874
(d) 2.3971
(e) 1.3321

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Department of Mathematics

CODE02 **CODE02**

Math 371
Final Exam
251
December 15, 2025
Net Time Allowed: 120 Minutes

Name	
ID	
Sec	

Check that this exam has 20 questions.

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7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. Consider the initial value problem

$$y' = y, \qquad 0 \leq t \leq 0.5, \qquad y(0) = 1,$$

whose exact solution is $y(t) = e^t$.

Apply Euler's method with step size $h = 0.25$ to obtain the approximation w_2 to $y(0.5)$ after two steps. Using the global error bound for Euler's method, an upper bound for $|y(0.5) - w_2|$ is

(a) $\frac{e-1}{8}$

(b) $\frac{e-\sqrt{e}}{16}$

(c) $\frac{\sqrt{e}-1}{8}$

(d) $\frac{e-\sqrt{e}}{4}$

(e) $\frac{e-\sqrt{e}}{8}$

2. If the quadratic Lagrange interpolating polynomial through the points $(0, 0)$, $(0.6, 0.6841)$ and $(1.2, 1.3133)$ is used to approximate $f(x)$, then $f(0.8) \approx$

(a) 0.9275

(b) 0.8999

(c) 0.8840

(d) 0.9121

(e) 0.8734

3. Using the Composite Simpson's rule with $n = 4$ subintervals to approximate

$$\int_0^2 x^2 \cos x \, dx,$$

we obtain

(a) 0.1320
(b) 0.1415
(c) 0.1550
(d) 0.1789
(e) 0.1632

4. Let $\{(1, 2, -1)^t,\ (2, 1, 3)^t\}$ be a set of two linearly independent vectors in $\mathbb{R}^3$. By the Gram–Schmidt process the orthogonal set is $\{(1, 2, -1)^t,\ V\}$. Then $V =$

(a) $(1, 0, 3)^t$
(b) $(11, 4, 19)^t$
(c) $(4, 11, 19)^t$
(d) $(5, 2, 7)^t$
(e) $(11, 4, -19)^t$

5. Let $x^{(0)} = (0, 0, 0)$. Consider applying the Gauss–Seidel method to the system

$$\begin{cases} 4x_1 - x_2 + x_3 = 7, \\ -x_1 + 3x_2 - 2x_3 = -4, \\ 2x_1 + x_2 + 5x_3 = 1. \end{cases}$$

Let $x^{(1)} = \left(x_1^{(1)}, x_2^{(1)}, x_3^{(1)}\right)$ and $x^{(2)} = \left(x_1^{(2)}, x_2^{(2)}, x_3^{(2)}\right)$ be the first two Gauss–Seidel iterates. Then $x_3^{(1)} + x_2^{(2)} \approx$

(a) -1.1833
(b) -1.2567
(c) -1.3667
(d) -1.2833
(e) -1.0167

6. Consider the initial value problem

$$y' = \cos(2t) + \sin(3t), \qquad 0 \le t \le 1, \qquad y(0) = 1,$$

whose exact solution is

$$y(t) = \frac{1}{2}\sin(2t) - \frac{1}{3}\cos(3t) + \frac{4}{3}.$$

Using the classical fourth-order Runge–Kutta method with step size $h = 0.5$ (two steps) to compute the approximation w_2 to $y(1)$, the actual error

$$|y(1) - w_2|$$

is approximately

(a) 1.41×10^{-3}
(b) 1.00×10^{-2}
(c) 5.00×10^{-4}
(d) 1.00×10^{-4}
(e) 5.00×10^{-3}

7. If $A = LU$, where

$$L = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{pmatrix}, \quad U \text{ is upper triangular, and} \quad A = \begin{pmatrix} 2 & 1 & -1 \\ 4 & 3 & 1 \\ 6 & 5 & 2 \end{pmatrix},$$

then $a + b + 2c =$

(a) 1
(b) 5
(c) 3
(d) 6
(e) 9

8. Let $\mathbf{x} = (x_1, x_2)^t$ and let $\mathbf{x}^{(1)}$ be the first iterate of the steepest descent method in solving the nonlinear system

$$\begin{cases} x_1^2 + 4x_2^2 = 5, \\ x_1 x_2 = 1, \end{cases}$$

Take the initial guess $\mathbf{x}^{(0)} = (1, \frac{1}{2})^t$ and step size $\alpha = 0.02$. Then $\mathbf{x}^{(1)}$ is approximately

(a) $(0.8750, 0.7500)^t$
(b) $(1.0500, 0.7500)^t$
(c) $(1.1250, 0.6500)^t$
(d) $(1.1250, 0.8500)^t$
(e) $(1.1250, 0.7500)^t$

9. Apply Newton's Method to approximate the x-value of the intersection point of the graphs of

$$f(x) = \ln(1 + x^2) \quad \text{and} \quad g(x) = 2 - e^{-x}.$$

Use $p_0 = 0.5$. Continue the iterations until $|p_N - p_{N-1}| < 0.4$. Then p_N is

(a) 2.3971
(b) 1.3321
(c) 1.6564
(d) 2.1874
(e) 4.9937

10. Let $\mathbf{x} = (x_1, x_2)^t$ and let $\mathbf{x}^{(1)}$ be the first iterate of Newton's method applied to the nonlinear system

$$\begin{cases} x_1^2 + x_2^2 - 5 = 0, \\ x_1^2 - x_2 - 1 = 0, \end{cases}$$

with initial guess $\mathbf{x}^{(0)} = (1, 1)^t$. Let $\|\cdot\|_2$ denote the Euclidean norm on $\mathbb{R}^2$. Then

$$\left\|\mathbf{x}^{(1)}\right\|_2^2 \approx$$

(a) 4.50
(b) 3.25
(c) 5.75
(d) 6.14
(e) 7.00

11. Given the following data

$$f(1.4) = 14.1, \qquad f(1.5) = 15.6, \qquad f(1.6) = 18.5,$$

use the three-point midpoint and endpoint formulas to approximate $f'(1.5)+f'(1.6)$. Then

$$f'(1.5) + f'(1.6) \approx$$

(a) 58
(b) 56
(c) 54
(d) 60
(e) 62

12. Let $P(x) = \frac{1}{5}x + b$ be the least squares polynomial of degree one for the data in the table

x_i	-1	0	1	α
y_i	2	-3	5	0

where $\alpha > 0$. Then $P(0) =$

(a) 0.9000
(b) 0.5000
(c) 1.2000
(d) 0.7500
(e) 1.0500

13. Let $P_2(x)$ be the second Taylor polynomial for $f(x) = \ln(x^2 + 3)$ about $x_0 = 0$. By Taylor's theorem, the smallest upper bound you can justify for $|f(1.5) - P_2(1.5)|$ is

(a) 5.00×10^{-2}
(b) 3.50×10^{-1}
(c) 2.50×10^{-1}
(d) 3.15×10^{-1}
(e) 1.00×10^{-1}

14. Let A be a real $m \times n$ matrix with $m \geq n$, and let

$$A = USV^t$$

be a singular value decomposition of A. Which of the following statements is *not always true*?

(a) The matrix S has nonnegative entries on its main diagonal and zeros elsewhere.
(b) The rank of A equals the number of nonzero diagonal entries of S.
(c) The columns of U corresponding to nonzero singular values form an orthonormal basis for the column space of A.
(d) If A is square and invertible, then all singular values of A are equal to 1.
(e) U and V care always invertible.

15. Let

$$A = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix},$$

with $a > b > c > 0$. If $(\rho(A) + \rho(A^T))\,\|A\|_\infty = 2k$, then $k =$

(a) $a^2 + 2ab$

(b) $a + b$

(c) $2a + 2ab$

(d) $a + ab^2$

(e) $a^2 + ab$

16. Consider $f(x) = 5x - 12\pi + 7e$. Assume three-digit rounding floating-point arithmetic is used, with rounding after every basic operation (each addition, subtraction, multiplication, or division). Use the three-digit approximations $\pi \approx 3.14$ and $e \approx 2.72$, and evaluate $f(x)$ at $x = 17/61$. In this arithmetic, $f(17/61)$ is approximately

(a) -17.2

(b) -17.4

(c) -17.5

(d) -17.1

(e) -17.3

17. Using Gaussian elimination with partial pivoting *and three-digit chopping arithmetic* to solve

$$\begin{cases} ex + \pi y = 1, \\ \pi x + ey = 0, \end{cases}$$

let $\mathbf{u} = (x, y)^T$ be the solution vector. Then $\|\mathbf{u}\|_\infty \approx$

(a) 1.23
(b) 1.06
(c) 2.43
(d) 2.76
(e) 2.05

18. Let

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & a \end{pmatrix}, \qquad a > 0,$$

be a 3×2 matrix. Suppose that 2 is a singular value of A. Then $a =$

(a) 2
(b) 1
(c) 4
(d) $\sqrt{2}$
(e) $\frac{1}{\sqrt{2}}$

19. Given the boundary-value problem

$$y'' = 9y, \qquad 0 \leq x \leq 1, \qquad y(0) = 0, \ y(1) = 0.$$

Using the linear finite difference method with step size $h = \dfrac{1}{3}$ to obtain a system

$$A\mathbf{w} = \mathbf{b}$$

for the interior approximations $\mathbf{w} = (w_1, w_2)^t$ (at $x_1 = \frac{1}{3}$ and $x_2 = \frac{2}{3}$), the sum of the diagonal entries of A is

(a) -4
(b) -2
(c) -6
(d) 2
(e) 0

20. Given the boundary-value problem

$$y'' = 2y' - y + x, \qquad 0 \leq x \leq 1, \qquad y(0) = 3, \ y(1) = 4.$$

Using the Linear Finite-Difference method with step size $h = 0.5$, $y(0.5) \approx$

(a) 3.90
(b) 3.75
(c) 3.64
(d) 3.25
(e) 3.50

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE03		CODE03

Math 371
Final Exam
251
December 15, 2025
Net Time Allowed: 120 Minutes

Name			
ID		**Sec**	

Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. Let

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & a \end{pmatrix}, \qquad a > 0,$$

be a 3×2 matrix. Suppose that 2 is a singular value of A. Then $a =$

(a) $\sqrt{2}$
(b) 4
(c) $\dfrac{1}{\sqrt{2}}$
(d) 2
(e) 1

2. Given the boundary-value problem

$$y'' = 9y, \qquad 0 \le x \le 1, \qquad y(0) = 0,\ y(1) = 0.$$

Using the linear finite difference method with step size $h = \dfrac{1}{3}$ to obtain a system

$$A\mathbf{w} = \mathbf{b}$$

for the interior approximations $\mathbf{w} = (w_1, w_2)^t$ (at $x_1 = \frac{1}{3}$ and $x_2 = \frac{2}{3}$), the sum of the diagonal entries of A is

(a) -2
(b) 2
(c) 0
(d) -6
(e) -4

3. Given the following data

$$f(1.4) = 14.1, \qquad f(1.5) = 15.6, \qquad f(1.6) = 18.5,$$

use the three-point midpoint and endpoint formulas to approximate $f'(1.5)+f'(1.6)$. Then

$$f'(1.5) + f'(1.6) \approx$$

(a) 56
(b) 58
(c) 54
(d) 60
(e) 62

4. Let $P_2(x)$ be the second Taylor polynomial for $f(x) = \ln(x^2 + 3)$ about $x_0 = 0$. By Taylor's theorem, the smallest upper bound you can justify for $|f(1.5) - P_2(1.5)|$ is

(a) 2.50×10^{-1}
(b) 3.15×10^{-1}
(c) 3.50×10^{-1}
(d) 1.00×10^{-1}
(e) 5.00×10^{-2}

5. Consider the initial value problem

$$y' = \cos(2t) + \sin(3t), \qquad 0 \le t \le 1, \qquad y(0) = 1,$$

whose exact solution is

$$y(t) = \frac{1}{2}\sin(2t) - \frac{1}{3}\cos(3t) + \frac{4}{3}.$$

Using the classical fourth-order Runge–Kutta method with step size $h = 0.5$ (two steps) to compute the approximation w_2 to $y(1)$, the actual error

$$|y(1) - w_2|$$

is approximately

(a) 1.00×10^{-4}
(b) 1.00×10^{-2}
(c) 5.00×10^{-4}
(d) 1.41×10^{-3}
(e) 5.00×10^{-3}

6. Consider $f(x) = 5x - 12\pi + 7e$. Assume three-digit rounding floating-point arithmetic is used, with rounding after every basic operation (each addition, subtraction, multiplication, or division). Use the three-digit approximations $\pi \approx 3.14$ and $e \approx 2.72$, and evaluate $f(x)$ at $x = 17/61$. In this arithmetic, $f(17/61)$ is approximately

(a) -17.4
(b) -17.2
(c) -17.5
(d) -17.1
(e) -17.3

7. Let A be a real $m \times n$ matrix with $m \geq n$, and let

$$A = USV^t$$

be a singular value decomposition of A. Which of the following statements is *not always true*?

(a) The rank of A equals the number of nonzero diagonal entries of S.

(b) U and V care always invertible.

(c) The columns of U corresponding to nonzero singular values form an orthonormal basis for the column space of A.

(d) If A is square and invertible, then all singular values of A are equal to 1.

(e) The matrix S has nonnegative entries on its main diagonal and zeros elsewhere.

8. Apply Newton's Method to approximate the x-value of the intersection point of the graphs of

$$f(x) = \ln(1 + x^2) \quad \text{and} \quad g(x) = 2 - e^{-x}.$$

Use $p_0 = 0.5$. Continue the iterations until $|p_N - p_{N-1}| < 0.4$. Then p_N is

(a) 2.1874

(b) 1.3321

(c) 2.3971

(d) 1.6564

(e) 4.9937

9. Using Gaussian elimination with partial pivoting *and three-digit chopping arithmetic* to solve

$$\begin{cases} ex + \pi y = 1, \\ \pi x + ey = 0, \end{cases}$$

let $\mathbf{u} = (x, y)^T$ be the solution vector. Then $\|\mathbf{u}\|_\infty \approx$

(a) 2.05
(b) 2.43
(c) 2.76
(d) 1.06
(e) 1.23

10. If $A = LU$, where

$$L = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{pmatrix}, \quad U \text{ is upper triangular, and} \quad A = \begin{pmatrix} 2 & 1 & -1 \\ 4 & 3 & 1 \\ 6 & 5 & 2 \end{pmatrix},$$

then $a + b + 2c =$

(a) 9
(b) 3
(c) 5
(d) 6
(e) 1

11. If the quadratic Lagrange interpolating polynomial through the points $(0,0)$, $(0.6, 0.6841)$ and $(1.2, 1.3133)$ is used to approximate $f(x)$, then $f(0.8) \approx$

    (a) 0.9275
    (b) 0.8734
    (c) 0.8999
    (d) 0.8840
    (e) 0.9121

12. Let $P(x) = \frac{1}{5}x + b$ be the least squares polynomial of degree one for the data in the table

x_i	-1	0	1	α
y_i	2	-3	5	0

where $\alpha > 0$. Then $P(0) =$

    (a) 0.5000
    (b) 0.7500
    (c) 1.0500
    (d) 0.9000
    (e) 1.2000

13. Given the boundary-value problem

$$y'' = 2y' - y + x, \qquad 0 \le x \le 1, \qquad y(0) = 3,\ y(1) = 4.$$

Using the Linear Finite-Difference method with step size $h = 0.5$, $y(0.5) \approx$

(a) 3.50
(b) 3.64
(c) 3.90
(d) 3.75
(e) 3.25

14. Let

$$A = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix},$$

with $a > b > c > 0$. If $(\rho(A) + \rho(A^T))\,\|A\|_\infty = 2k$, then $k =$

(a) $a + b$
(b) $a^2 + ab$
(c) $a + ab^2$
(d) $a^2 + 2ab$
(e) $2a + 2ab$

15. Consider the initial value problem

$$y' = y, \qquad 0 \le t \le 0.5, \qquad y(0) = 1,$$

whose exact solution is $y(t) = e^t$.

Apply Euler's method with step size $h = 0.25$ to obtain the approximation w_2 to $y(0.5)$ after two steps. Using the global error bound for Euler's method, an upper bound for $|y(0.5) - w_2|$ is

(a) $\dfrac{e - \sqrt{e}}{4}$

(b) $\dfrac{e - \sqrt{e}}{16}$

(c) $\dfrac{e - \sqrt{e}}{8}$

(d) $\dfrac{\sqrt{e} - 1}{8}$

(e) $\dfrac{e - 1}{8}$

16. Using the Composite Simpson's rule with $n = 4$ subintervals to approximate

$$\int_0^2 x^2 \cos x \, dx,$$

we obtain

(a) 0.1415

(b) 0.1550

(c) 0.1320

(d) 0.1789

(e) 0.1632

17. Let $\{(1,2,-1)^t,\ (2,1,3)^t\}$ be a set of two linearly independent vectors in $\mathbb{R}^3$. By the Gram–Schmidt process the orthogonal set is $\{(1,2,-1)^t,\ V\}$. Then $V =$

(a) $(4,11,19)^t$
(b) $(11,4,-19)^t$
(c) $(1,0,3)^t$
(d) $(11,4,19)^t$
(e) $(5,2,7)^t$

18. Let $\mathbf{x} = (x_1, x_2)^t$ and let $\mathbf{x}^{(1)}$ be the first iterate of Newton's method applied to the nonlinear system

$$\begin{cases} x_1^2 + x_2^2 - 5 = 0, \\ x_1^2 - x_2 - 1 = 0, \end{cases}$$

with initial guess $\mathbf{x}^{(0)} = (1,1)^t$. Let $\|\cdot\|_2$ denote the Euclidean norm on $\mathbb{R}^2$. Then

$$\left\|\mathbf{x}^{(1)}\right\|_2^2 \approx$$

(a) 4.50
(b) 6.14
(c) 7.00
(d) 3.25
(e) 5.75

19. Let $\mathbf{x} = (x_1, x_2)^t$ and let $\mathbf{x}^{(1)}$ be the first iterate of the steepest descent method in solving the nonlinear system

$$\begin{cases} x_1^2 + 4x_2^2 = 5, \\ x_1 x_2 = 1, \end{cases}$$

Take the initial guess $\mathbf{x}^{(0)} = (1, \frac{1}{2})^t$ and step size $\alpha = 0.02$. Then $\mathbf{x}^{(1)}$ is approximately

(a) $(1.1250, 0.8500)^t$
(b) $(0.8750, 0.7500)^t$
(c) $(1.0500, 0.7500)^t$
(d) $(1.1250, 0.6500)^t$
(e) $(1.1250, 0.7500)^t$

20. Let $x^{(0)} = (0, 0, 0)$. Consider applying the Gauss–Seidel method to the system

$$\begin{cases} 4x_1 - x_2 + x_3 = 7, \\ -x_1 + 3x_2 - 2x_3 = -4, \\ 2x_1 + x_2 + 5x_3 = 1. \end{cases}$$

Let $x^{(1)} = \left(x_1^{(1)}, x_2^{(1)}, x_3^{(1)}\right)$ and $x^{(2)} = \left(x_1^{(2)}, x_2^{(2)}, x_3^{(2)}\right)$ be the first two Gauss–Seidel iterates. Then $x_3^{(1)} + x_2^{(2)} \approx$

(a) -1.0167
(b) -1.3667
(c) -1.2833
(d) -1.1833
(e) -1.2567

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE04 **CODE04**

Math 371
Final Exam
251
December 15, 2025
Net Time Allowed: 120 Minutes

Name			
ID		**Sec**	

Check that this exam has 20 questions.

Important Instructions:

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2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
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6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. Using Gaussian elimination with partial pivoting *and three-digit chopping arithmetic* to solve

$$\begin{cases} ex + \pi y = 1, \\ \pi x + ey = 0, \end{cases}$$

let $\mathbf{u} = (x, y)^T$ be the solution vector. Then $\|\mathbf{u}\|_\infty \approx$

(a) 2.05

(b) 2.43

(c) 2.76

(d) 1.06

(e) 1.23

2. Let $x^{(0)} = (0, 0, 0)$. Consider applying the Gauss–Seidel method to the system

$$\begin{cases} 4x_1 - x_2 + x_3 = 7, \\ -x_1 + 3x_2 - 2x_3 = -4, \\ 2x_1 + x_2 + 5x_3 = 1. \end{cases}$$

Let $x^{(1)} = \left(x_1^{(1)}, x_2^{(1)}, x_3^{(1)}\right)$ and $x^{(2)} = \left(x_1^{(2)}, x_2^{(2)}, x_3^{(2)}\right)$ be the first two Gauss–Seidel iterates. Then $x_3^{(1)} + x_2^{(2)} \approx$

(a) -1.2833

(b) -1.0167

(c) -1.1833

(d) -1.3667

(e) -1.2567

3. Consider the initial value problem

$$y' = y, \qquad 0 \le t \le 0.5, \qquad y(0) = 1,$$

whose exact solution is $y(t) = e^t$.

Apply Euler's method with step size $h = 0.25$ to obtain the approximation w_2 to $y(0.5)$ after two steps. Using the global error bound for Euler's method, an upper bound for $|y(0.5) - w_2|$ is

(a) $\dfrac{e - \sqrt{e}}{8}$

(b) $\dfrac{e - \sqrt{e}}{16}$

(c) $\dfrac{\sqrt{e} - 1}{8}$

(d) $\dfrac{e - 1}{8}$

(e) $\dfrac{e - \sqrt{e}}{4}$

4. Let $P_2(x)$ be the second Taylor polynomial for $f(x) = \ln(x^2 + 3)$ about $x_0 = 0$. By Taylor's theorem, the smallest upper bound you can justify for $|f(1.5) - P_2(1.5)|$ is

(a) 1.00×10^{-1}

(b) 2.50×10^{-1}

(c) 3.15×10^{-1}

(d) 3.50×10^{-1}

(e) 5.00×10^{-2}

5. Let A be a real $m \times n$ matrix with $m \geq n$, and let

$$A = USV^t$$

be a singular value decomposition of A. Which of the following statements is *not always true*?

(a) The columns of U corresponding to nonzero singular values form an orthonormal basis for the column space of A.

(b) U and V care always invertible.

(c) The rank of A equals the number of nonzero diagonal entries of S.

(d) If A is square and invertible, then all singular values of A are equal to 1.

(e) The matrix S has nonnegative entries on its main diagonal and zeros elsewhere.

6. Given the boundary-value problem

$$y'' = 9y, \qquad 0 \leq x \leq 1, \qquad y(0) = 0,\ y(1) = 0.$$

Using the linear finite difference method with step size $h = \dfrac{1}{3}$ to obtain a system

$$A\mathbf{w} = \mathbf{b}$$

for the interior approximations $\mathbf{w} = (w_1, w_2)^t$ (at $x_1 = \frac{1}{3}$ and $x_2 = \frac{2}{3}$), the sum of the diagonal entries of A is

(a) -2

(b) -4

(c) -6

(d) 2

(e) 0

7. Using the Composite Simpson's rule with $n = 4$ subintervals to approximate

$$\int_0^2 x^2 \cos x \, dx,$$

we obtain

(a) 0.1415
(b) 0.1789
(c) 0.1550
(d) 0.1320
(e) 0.1632

8. Given the following data

$$f(1.4) = 14.1, \qquad f(1.5) = 15.6, \qquad f(1.6) = 18.5,$$

use the three-point midpoint and endpoint formulas to approximate $f'(1.5) + f'(1.6)$. Then

$$f'(1.5) + f'(1.6) \approx$$

(a) 60
(b) 58
(c) 62
(d) 54
(e) 56

9. If the quadratic Lagrange interpolating polynomial through the points $(0,0)$, $(0.6, 0.6841)$ and $(1.2, 1.3133)$ is used to approximate $f(x)$, then $f(0.8) \approx$

(a) 0.8999
(b) 0.8840
(c) 0.8734
(d) 0.9121
(e) 0.9275

10. Apply Newton's Method to approximate the x-value of the intersection point of the graphs of

$$f(x) = \ln(1 + x^2) \quad \text{and} \quad g(x) = 2 - e^{-x}.$$

Use $p_0 = 0.5$. Continue the iterations until $|p_N - p_{N-1}| < 0.4$. Then p_N is

(a) 1.6564
(b) 1.3321
(c) 2.3971
(d) 2.1874
(e) 4.9937

11. Given the boundary-value problem

$$y'' = 2y' - y + x, \qquad 0 \le x \le 1, \qquad y(0) = 3,\ y(1) = 4.$$

Using the Linear Finite-Difference method with step size $h = 0.5$, $y(0.5) \approx$

(a) 3.25
(b) 3.90
(c) 3.64
(d) 3.50
(e) 3.75

12. Let $P(x) = \frac{1}{5}x + b$ be the least squares polynomial of degree one for the data in the table

x_i	-1	0	1	α
y_i	2	-3	5	0

where $\alpha > 0$. Then $P(0) =$

(a) 1.2000
(b) 0.5000
(c) 0.9000
(d) 1.0500
(e) 0.7500

13. Let

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & a \end{pmatrix}, \qquad a > 0,$$

be a 3×2 matrix. Suppose that 2 is a singular value of A. Then $a =$

(a) $\dfrac{1}{\sqrt{2}}$

(b) $\sqrt{2}$

(c) 1

(d) 2

(e) 4

14. Consider the initial value problem

$$y' = \cos(2t) + \sin(3t), \qquad 0 \le t \le 1, \qquad y(0) = 1,$$

whose exact solution is

$$y(t) = \frac{1}{2}\sin(2t) - \frac{1}{3}\cos(3t) + \frac{4}{3}.$$

Using the classical fourth-order Runge–Kutta method with step size $h = 0.5$ (two steps) to compute the approximation w_2 to $y(1)$, the actual error

$$|y(1) - w_2|$$

is approximately

(a) 1.00×10^{-4}

(b) 5.00×10^{-3}

(c) 5.00×10^{-4}

(d) 1.41×10^{-3}

(e) 1.00×10^{-2}

15. Consider $f(x) = 5x - 12\pi + 7e$. Assume three-digit rounding floating-point arithmetic is used, with rounding after every basic operation (each addition, subtraction, multiplication, or division). Use the three-digit approximations $\pi \approx 3.14$ and $e \approx 2.72$, and evaluate $f(x)$ at $x = 17/61$. In this arithmetic, $f(17/61)$ is approximately

(a) -17.2
(b) -17.3
(c) -17.4
(d) -17.5
(e) -17.1

16. If $A = LU$, where

$$L = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{pmatrix}, \quad U \text{ is upper triangular, and} \quad A = \begin{pmatrix} 2 & 1 & -1 \\ 4 & 3 & 1 \\ 6 & 5 & 2 \end{pmatrix},$$

then $a + b + 2c =$

(a) 5
(b) 3
(c) 6
(d) 9
(e) 1

17. Let

$$A = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix},$$

with $a > b > c > 0$. If $(\rho(A) + \rho(A^T))\, \|A\|_\infty = 2k$, then $k =$

(a) $a^2 + ab$

(b) $a + b$

(c) $a^2 + 2ab$

(d) $2a + 2ab$

(e) $a + ab^2$

18. Let $\{(1, 2, -1)^t,\ (2, 1, 3)^t\}$ be a set of two linearly independent vectors in $\mathbb{R}^3$. By the Gram–Schmidt process the orthogonal set is $\{(1, 2, -1)^t,\ V\}$. Then $V =$

(a) $(11, 4, -19)^t$

(b) $(4, 11, 19)^t$

(c) $(11, 4, 19)^t$

(d) $(5, 2, 7)^t$

(e) $(1, 0, 3)^t$

19. Let $\mathbf{x} = (x_1, x_2)^t$ and let $\mathbf{x}^{(1)}$ be the first iterate of the steepest descent method in solving the nonlinear system

$$\begin{cases} x_1^2 + 4x_2^2 = 5, \\ x_1 x_2 = 1, \end{cases}$$

Take the initial guess $\mathbf{x}^{(0)} = (1, \frac{1}{2})^t$ and step size $\alpha = 0.02$. Then $\mathbf{x}^{(1)}$ is approximately

(a) $(0.8750, 0.7500)^t$
(b) $(1.1250, 0.7500)^t$
(c) $(1.0500, 0.7500)^t$
(d) $(1.1250, 0.8500)^t$
(e) $(1.1250, 0.6500)^t$

20. Let $\mathbf{x} = (x_1, x_2)^t$ and let $\mathbf{x}^{(1)}$ be the first iterate of Newton's method applied to the nonlinear system

$$\begin{cases} x_1^2 + x_2^2 - 5 = 0, \\ x_1^2 - x_2 - 1 = 0, \end{cases}$$

with initial guess $\mathbf{x}^{(0)} = (1, 1)^t$. Let $\|\cdot\|_2$ denote the Euclidean norm on $\mathbb{R}^2$. Then

$$\left\|\mathbf{x}^{(1)}\right\|_2^2 \approx$$

(a) 3.25
(b) 4.50
(c) 6.14
(d) 5.75
(e) 7.00

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	E 20	E 8	A 14	E 7
2	A	B 4	B 4	D 18	D 11
3	A	B 17	C 6	B 5	A 8
4	A	D 5	B 13	B 1	C 1
5	A	E 18	C 11	D 9	D 15
6	A	D 8	A 9	E 2	C 18
7	A	A 15	E 20	D 15	C 6
8	A	A 10	E 17	C 3	B 5
9	A	C 12	A 3	E 7	A 4
10	A	C 1	D 16	A 20	C 3
11	A	B 13	A 5	C 4	C 19
12	A	C 11	A 12	D 12	C 12
13	A	B 6	D 1	B 19	B 14
14	A	C 7	D 15	B 10	D 9
15	A	E 2	E 10	C 8	B 2
16	A	B 14	E 2	B 6	D 20
17	A	A 16	A 7	D 13	A 10
18	A	E 9	D 14	B 16	C 13
19	A	C 19	C 18	E 17	B 17
20	A	D 3	C 19	B 11	C 16

Answer Counts

V	A	B	C	D	E
1	3	5	5	3	4
2	5	2	4	4	5
3	2	7	3	5	3
4	3	4	8	4	1