King Fahd University of Petroleum and Minerals

Department of Mathematics

Math 371

Major Exam I

251

Sep 28 , 2025

Net Time Allowed: 90 Minutes

USE THIS AS A TEMPLATE

Write your questions, once you are satisfied upload this file.

1. Let $P_2(x)$ be the second Taylor polynomial about 0 for $f(x) = \ln(1+x)$. Give an upper bound for the error $|f(0.3) - P_2(0.3)|$ from Taylor's remainder.

   (a) 0.00900
   (b) 0.00750
   (c) 0.01050
   (d) 0.00450
   (e) 0.01200

2. Using *three-digit chopping* arithmetic, compute

$$-9\pi + 4e - \frac{5}{128}.$$

   (a) -17.4
   (b) -17.3
   (c) -17.2
   (d) -17.5
   (e) -17.0

3. Suppose p^* approximates π with relative error at most 2×10^{-3}. The largest interval in which p^* must lie is

   (a) $[3.13531,\ 3.14788]$
   (b) $[3.12000,\ 3.16000]$
   (c) $(3.10000,\ 3.18000)$
   (d) $(3.11000,\ 3.15000)$
   (e) $(3.00000,\ 3.20000)$

4. Apply the Bisection method to $f(x) = x^3 - 2$ on $[1, 2]$. After the first three approximations p_1, p_2, p_3, the sum $p_1 + p_2 + p_3$ equals

   (a) 4.125
   (b) 3.750
   (c) 4.000
   (d) 3.625
   (e) 4.250

5. Using Bisection method on $[0, 2]$, the minimum number of iterations to achieve an accuracy within 10^{-4} is

   (a) $n = 15$
   (b) $n = 14$
   (c) $n = 12$
   (d) $n = 10$
   (e) $n = 8$

6. Use fixed-point iteration with $g(x) = (2 + x)^{1/3}$ and $p_0 = 1.2$ to solve $x^3 - x - 2 = 0$ to within 10^{-2} accuracy. The resulting approximation is

   (a) 1.520
   (b) 1.505
   (c) 1.540
   (d) 1.495
   (e) 1.560

7. Consider $g(x) = 1 + \frac{1}{5}\sin x$ on $[0, 2]$. Then $|g'(x)| \leq \frac{1}{5}$. With $p_0 = 0$, the minimum number of iterations such that $|p_n - p^*| \leq 10^{-2}$ is

   (a) $n = 4$
   (b) $n = 5$
   (c) $n = 6$
   (d) $n = 7$
   (e) $n = 3$

8. Apply Newton's method to $f(x) = x - \cos x$ with $p_0 = 0.5$. After three iterations, $p_3 \approx$

   (a) 0.73909
   (b) 0.70711
   (c) 0.69315
   (d) 0.74000
   (e) 0.73600

9. The Secant method for $f(x) = e^x - 2$ with $p_0 = 0$ and $p_1 = 1$ gives $p_3 \approx$

   (a) 0.67669
   (b) 0.69315
   (c) 0.65000
   (d) 0.70000
   (e) 0.66000

10. Let P_2 be the quadratic Lagrange interpolant of $f(x) = \ln(1 + x)$ at the nodes $x_0 = 0$, $x_1 = 0.5$, $x_2 = 1.0$. The coefficient of x^2 in $P_2(x)$ is approximately

    (a) -0.235566
    (b) -0.210000
    (c) -0.180000
    (d) -0.250000
    (e) -0.200000

11. A natural cubic spline S on $[0,2]$ is

$$S(x) = \begin{cases} 2 - x + x^3, & 0 \le x \le 1, \\ \alpha + b(x-1) + c(x-1)^2 + d(x-1)^3, & 1 \le x \le 2. \end{cases}$$

Then $b + c + d$ equals

(a) 4

(b) 0

(c) -1

(d) 2

(e) -3

12. Using the forward-difference formula with step $h = 0.02$ at $x_0 = 1.6$ for $f(x) = e^{-x}$, approximate $f'(x_0)$.

(a) -0.199891

(b) -0.201897

(c) -0.198000

(d) -0.205000

(e) -0.196500

13. Using the composite Trapezoidal rule with $n = 4$, approximate $\int_0^2 \cos(x^2)\,dx$.

(a) 0.5271
(b) 0.5200
(c) 0.5342
(d) 0.5300
(e) 0.5526

14. The minimum value of n so that the composite Simpson rule approximates $\int_1^2 \ln x\,dx$ within 10^{-5} accuracy is

(a) $n = 8$
(b) $n = 10$
(c) $n = 12$
(d) $n = 6$
(e) $n = 14$