King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 371

Major Exam II

251

Nov 9 , 2025

Net Time Allowed: 90 Minutes

USE THIS AS A TEMPLATE

Write your questions, once you are satisfied upload this file.

1. Which of the following functions does **not** satisfy a Lipschitz condition on the domain $D = \{(t, y) \,|\, 0 \le t \le 1,\ 0 \le y \le 2\}$?

   (a) $f(t, y) = \sqrt{y} + t\sin(2y)$
   (b) $f(t, y) = t + y$
   (c) $f(t, y) = y^2 + t^3$
   (d) $f(t, y) = t^2 e^y$
   (e) $f(t, y) = 1 + t^3 y$

2. Using Euler's method with $h = 0.2$ to solve $y' = te^{2t} - 2y,\ y(0) = 1$ on $t \in [0, 1]$, then $y(0.4) \approx$

   (a) 0.420
   (b) 0.553
   (c) 0.612
   (d) 0.702
   (e) 0.846

3. For the IVP $y' = 1 + \frac{y}{t}$, $y(1) = 1$, the exact solution is $y(t) = t(1 + \ln t)$. Using Euler's method with $h = 0.25$ on $t \in [1, 1.5]$, the least upper bound for $|y(1.5) - w_2|$ is

(a) 0.0811
(b) 0.0750
(c) 0.0900
(d) 0.0700
(e) 0.1000

4. Using the midpoint method with $h = 0.2$ for $y' = t - y^2$, $y(0) = 1$ on $t \in [0, 1]$, then $y(0.4) \approx$

(a) 0.789
(b) 0.805
(c) 0.772
(d) 0.820
(e) 0.740

5. Using the RK4 method with $h = 0.1$ for $y' = y - t^2 + 1,\ y(0) = 0.5$ on $t \in [0, 1]$, then $y(0.1) \approx$

(a) 0.657
(b) 0.648
(c) 0.666
(d) 0.640
(e) 0.674

6. Consider the system

$$\begin{cases} 2x_1 + 4x_2 + x_3 = 5, \\ 2x_1 + x_2 + (\alpha - 5)\, x_3 = 1, \\ 3x_2 + (\alpha - 4)\, x_3 = 3. \end{cases}$$

This system has a unique solution for

(a) all real α except $\alpha = 5$
(b) only $\alpha = 5$
(c) all real α
(d) only $\alpha = 0$
(e) only $\alpha = -5$

7. If Gaussian elimination with partial pivoting and three-digit rounding arithmetic is used to solve the system

$$\begin{cases} 0.031x_1 + 58.1x_2 = 59.0, \\ 5.97x_1 - 6.22x_2 = 47.8, \end{cases}$$

then $x_1 + x_2 =$

(a) 10.07
(b) 9.95
(c) 10.10
(d) 9.80
(e) 10.25

8. The matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

is transformed by two permutations so that

$$P_2P_1A = \begin{bmatrix} a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{11} & a_{12} & a_{13} \end{bmatrix}.$$

The permutation matrices are

(a) $P_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(b) $P_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad P_2 = I$

(c) $P_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(d) $P_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(e) $P_1 = P_2 = I$

9. The matrix

$$A = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

has eigenvalues 7, 3, 2. Which of the following is an eigenvector corresponding to the largest eigenvalue?

(a) $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

(e) $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

10. Let

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 4 & 5 & 2 \\ 6 & 3 & 4 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 7 \\ 8 \end{bmatrix}.$$

Suppose $A = LU$ where $L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$. If $Ly = b$ is solved for $y = (y_1, y_2, y_3)^T$, then $y_1 + y_2 + y_3 =$

(a) 3.00
(b) 3.50
(c) 2.50
(d) 4.00
(e) 2.00

11. If the system

$$\begin{cases} 10x_1 - x_2 = 9, \\ -x_1 + 10x_2 - 2x_3 = 8, \\ -2x_2 + 10x_3 = 7, \end{cases}$$

is solved by the Jacobi method with $x^{(0)} = (1,1,1)^T$, then $x_1^{(2)} + x_2^{(2)} + x_3^{(2)} =$

(a) 3.01
(b) 2.90
(c) 2.75
(d) 3.10
(e) 2.60

12. Solve by Gauss–Seidel (two iterations) the system

$$\begin{cases} 4x_1 + x_2 = 5, \\ -x_1 + 5x_2 + x_3 = 10, \\ x_2 + 6x_3 = 12, \end{cases} \qquad \text{with} \quad x^{(0)} = (0,0,0)^T.$$

Then $x_2^{(2)} =$

(a) 1.813
(b) 1.750
(c) 1.900
(d) 1.700
(e) 1.600

13. Consider the linear system

$$\begin{cases} \frac{1}{3}x_1 + \frac{1}{4}x_2 = \frac{1}{40}, \\ \frac{1}{4}x_1 + \frac{1}{5}x_2 = \frac{1}{60}. \end{cases}$$

The exact solution is $x = (\frac{1}{6}, -\frac{1}{9})^T$. If an approximate solution $\tilde{x} = (0.165, -0.100)^T$ is used, compute $\|A\tilde{x} - b\|_\infty$.

(a) 0.0050
(b) 0.0046
(c) 0.0042
(d) 0.0054
(e) 0.0040

14. For $A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$, $\|A\|_2 =$

(a) 3.703
(b) 3.300
(c) 3.000
(d) 2.500
(e) 4.000