King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 371

Final Exam

251

May 25 , 2025

Net Time Allowed: 120 Minutes

MASTER VERSION

1. If $P_2(x)$ is the second Taylor polynomial of $f(x) = \ln(1+x)$ about $x = 0$, then $P_2(0.7) =$

   (a) 0.455
   (b) 0.460
   (c) 0.450
   (d) 0.430
   (e) 0.480

2. Consider the equation

   $$f(x) = x - 1 - \sin x = 0.$$

   Using the Bisection method on $[1, 2]$, $p_3 =$

   (a) 1.8750
   (b) 1.7500
   (c) 1.9375
   (d) 1.8125
   (e) 1.6875

3. Consider the equation

$$f(x) = e^x - 2 = 0.$$

Starting with $p_0 = 0.5$, find p_1 using Newton's method and then p_2 using Secant method, $p_2 =$

(a) 0.6912
(b) 0.6800
(c) 0.7000
(d) 0.7020
(e) 0.7100

4. Let $P_2(x)$ be the second Lagrange interpolating polynomial for $f(x) = \ln(0.5x + 1)$ on $[0, 2]$ using the nodes $x_0 = 0$, $x_1 = 1$, $x_2 = 2$. Then $P_2(1.2) \approx$

(a) 0.4724
(b) 0.4800
(c) 0.4600
(d) 0.4900
(e) 0.5000

5. Let $f(x) = x^3 - 2x$. Using the most accurate three-point formula with $h = 0.5$ to approximate $f'(1)$, then $f'(1) \approx$

   (a) 1.25
   (b) 1.00
   (c) 0.75
   (d) 1.50
   (e) 0.50

6. Use the composite Simpson method with $n = 4$ subintervals to approximate

$$\int_0^1 \frac{1}{x+3}\, dx.$$

   The approximation is

   (a) 0.2877
   (b) 0.2810
   (c) 0.2950
   (d) 0.2700
   (e) 0.3050

7. Consider the initial value problem

$$y' = t + y, \quad 0 \leq t \leq 0.5, \qquad y(0) = 1.$$

Using the midpoint method with step size $h = 0.25$, the approximation to $y(0.5)$ is

(a) 1.783
(b) 1.750
(c) 1.700
(d) 1.800
(e) 1.650

8. Use one iteration of the Gauss–Seidel method to solve the linear system

$$\begin{cases} 3x_1 - x_2 = 5, \\ -x_1 + 4x_2 - x_3 = 6, \\ -x_2 + 5x_3 = 5, \end{cases}$$

starting from $\mathbf{x}^{(0)} = (0,0,0)^t$. Then $\mathbf{x}^{(1)} \approx$

(a) $(1.667, 1.917, 1.383)^t$
(b) $(1.600, 1.800, 1.300)^t$
(c) $(1.500, 2.000, 1.400)^t$
(d) $(1.700, 1.900, 1.500)^t$
(e) $(1.600, 2.000, 1.500)^t$

9. Suppose

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 3 & 3 \\ 2 & 1 & 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 4 \\ 10 \\ 6 \end{bmatrix}.$$

Assume $A = LU$ is the Doolittle factorization of A, and consider $Ly = \mathbf{b}$, where $y = (y_1, y_2, y_3)^t$. Then $y_1 + y_2 + y_3 =$

(a) 8
(b) 6
(c) 7
(d) 9
(e) 5

10. Consider the matrix

$$A = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}.$$

The sum of the singular values of A is

(a) 8
(b) 6
(c) 4
(d) 10
(e) 12

11. Consider the data points $(0, 1.1)$, $(1, 1.9)$, $(3, 3.8)$. Let $P_1(x) = a_1x + a_0$ be the least squares polynomial of degree one that fits these data. Then $P_1(2) \approx$

(a) 2.871
(b) 2.500
(c) 3.100
(d) 2.700
(e) 3.300

12. Let $\mathbf{v}_1 = (1, 1, 0)^t$, $\mathbf{v}_2 = (1, 0, 1)^t$, and $\mathbf{v}_3 = (0, 1, 1)^t$ be linearly independent vectors in $\mathbb{R}^3$. Using the Gram–Schmidt process with $u_1 = \mathbf{v}_1$ to obtain orthogonal vectors u_1, u_2, u_3, the vector u_2 can be chosen as

(a) $(1, -1, 2)^t$
(b) $(1, 1, 0)^t$
(c) $(1, 0, 1)^t$
(d) $(0, 1, 1)^t$
(e) $(2, 0, 1)^t$

13. Which of the following matrices is orthogonal?

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \quad C = \frac{1}{2}\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}.$$

(a) A only

(b) B only

(c) C only

(d) A and C

(e) B and C

14. Consider the system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

and initial guess $\mathbf{x}^{(0)} = (0, 0)^t$. Using the Conjugate Gradient method with two-digit rounding arithmetic, the first iterate $\mathbf{x}^{(1)}$ is

(a) $(0.25, 0.50)^t$

(b) $(0.20, 0.40)^t$

(c) $(0.30, 0.60)^t$

(d) $(0.25, 0.40)^t$

(e) $(0.30, 0.50)^t$

15. Let

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

have the singular value decomposition $A = USV^t$, where

$$S = \begin{bmatrix} 2 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad V = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Then the first two columns u_1, u_2 of U are

(a) $u_1 = (1,0,0)^t,\ u_2 = (0,1,0)^t$
(b) $u_1 = (0,1,0)^t,\ u_2 = (1,0,0)^t$
(c) $u_1 = (0,0,1)^t,\ u_2 = (1,0,0)^t$
(d) $u_1 = (0,1,0)^t,\ u_2 = (0,0,1)^t$
(e) $u_1 = (1,0,0)^t,\ u_2 = (0,0,1)^t$

16. Consider the nonlinear system

$$\begin{cases} x_1^2 - x_2 = 0, \\ x_1 + x_2^2 = 3. \end{cases}$$

Using Newton's method with initial guess $\mathbf{x}^{(0)} = (1,1)^t$, the first iterate $\mathbf{x}^{(1)} = (x_1^{(1)}, x_2^{(1)})^t$ is

(a) $(1.2, 1.4)^t$
(b) $(0.8, 1.2)^t$
(c) $(1.0, 1.5)^t$
(d) $(1.4, 1.2)^t$
(e) $(1.3, 1.3)^t$

17. Consider the nonlinear system

$$\begin{cases} x_1^2 + x_2 - 1 = 0, \\ x_1 + x_2^2 - 1 = 0. \end{cases}$$

Using Steepest Descent method with $\alpha = 0.25$ and $x^{(0)} = (0,0)^t$. Then the second iterate $\mathbf{x}^{(2)}$ is approximately

(a) $(0.750, 0.750)^t$
(b) $(0.250, 0.250)^t$
(c) $(0.375, 0.375)^t$
(d) $(0.600, 0.600)^t$
(e) $(0.450, 0.450)^t$

18. In applying the linear Finite Difference method with step size $h = \frac{1}{4}$ to the boundary value problem

$$y'' - y = 0, \quad 0 \le x \le 1, \quad y(0) = 0, \ y(1) = 0,$$

a linear system of equations $A\mathbf{w} = \mathbf{b}$ is obtained. The sum of the diagonal elements of A is

(a) $\dfrac{99}{16}$
(b) $\dfrac{33}{16}$
(c) $\dfrac{9}{4}$
(d) 6
(e) $\dfrac{15}{4}$

19. If the Linear Finite Difference method is used to approximate the solution of the boundary value problem

$$y'' = y, \qquad 0 \le x \le 1, \qquad y(0) = 0, \qquad y(1) = \sin(1),$$

with step size $h = \frac{1}{2}$, then $y(0.5) \approx$

(a) 0.3740
(b) 0.3400
(c) 0.2600
(d) 0.2850
(e) 0.4100

20. Consider the following data:

x	1.0	2.0	3.0
y	2.3551	c	3.1116

Suppose the least squares exponential curve fitted to the data is

$$f(x) = 2.1021\, e^{b_1 x}.$$

Then the value of b_1 is

(a) 0.1392
(b) 1.1495
(c) -1.9711
(d) 0.3782
(e) -0.9723