King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 371

Final Exam

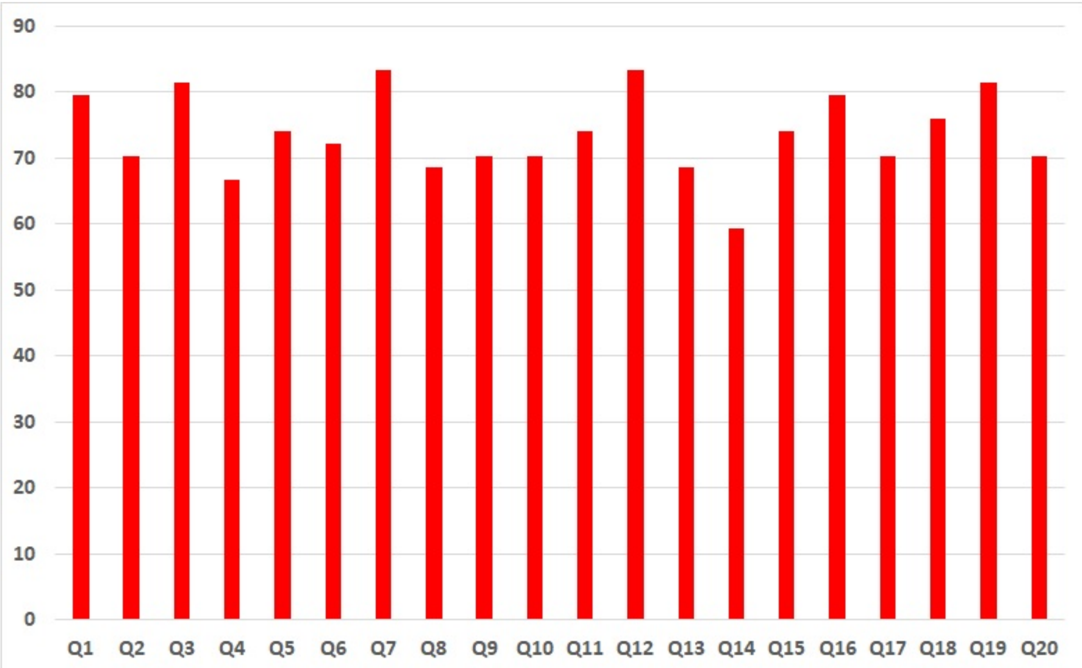
242

May 25 , 2025

Net Time Allowed: 120 Minutes

MASTER VERSION

Q	Average
Q1	79.6296
Q2	70.3704
Q3	81.4815
Q4	66.6667
Q5	74.0741
Q6	72.2222
Q7	83.3333
Q8	68.5185
Q9	70.3704
Q10	70.3704
Q11	74.0741
Q12	83.3333
Q13	68.5185
Q14	59.2593
Q15	74.0741
Q16	79.6296
Q17	70.3704
Q18	75.9259
Q19	81.4815
Q20	70.3704



Number of students = 54

King Fahd University of Petroleum and Minerals
Information Technology Center
Computing Services Section

Dept. of: Math
Course: MATH371 Final Exam
Semseter: 242

Monday, May 26, 2025

			Raw Score	% Score
Total no. of Students	54	**Course Mean:**	73.7	73.7
Course Std. Dev. :	24.29	**Max. Score:**	100	100
		Min. Score:	15	15

1. If $P_2(x)$ is the second Taylor polynomial of $f(x) = e^x \cos(x)$ about $x = 0$, then $P_2(0.3) =$

(a) 1.30
(b) 1.20
(c) 1.25
(d) 1.35
(e) 1.40

Avg:

79.6296

2. The vectors $\mathbf{v}_1 = (2, -1)^t$, $\mathbf{v}_2 = (1, 1)^t$, and $\mathbf{v}_3 = (1, 3)^t$ are

(a) Linearly dependent and not orthogonal
(b) Linearly dependent and orthogonal
(c) Linearly independent and not orthogonal
(d) Linearly independent and orthogonal
(e) Linearly independent and orthonormal

Avg:

70.3704

70.3704

3. Consider the equation $x^3 - 2x^2 - 5 = 0$. Use Newton's method with $p_0 = 2.7$ to find solution accurate to within 10^{-4}:

(a) 2.6906
(b) 2.7015
(c) 2.7105
(d) 2.6837
(e) 2.7200

Avg:

81.4815

4. If $P_2(x)$ is the second Lagrange polynomial that passes through the points (0, 0), (0.6, 0.47), (0.9, 0.6419). Then $P_2(0.45) =$

Avg:

66.6667

(a) 0.3683
(b) 0.3701
(c) 0.3803
(d) 0.3854
(e) 0.3602

5. Consider the natural cubic spline function:

$$S(x) = \begin{cases} s_1(x) = 1 + x, & -1 \le x \le 1, \\ s_2(x) = \frac{1}{8}x^3 + ax^2 + bx + c, & 1 \le x \le 2. \end{cases}$$

Then the value of $a + b + c =$

(a) 1.875

(b) 1.500

(c) 2.00

(d) 2.785

(e) 1.625

Avg:

74.0741

6. Use the composite Trapezoidal method with $n = 6$ to approximate the integral $\int_0^\pi x^2 \cos(x)dx$:

(a) -6.4287

(b) -6.1362

(c) -5.5381

(d) -5.9103

(e) -5.1024

Avg:

72.2222

7. Using Gaussain elimination with partial pivoting and three-digit chopping arithmetic for solving the linear system $\begin{cases} 0.03x_1 + 58.9x_2 = 59.2, \\ 5.31x_1 - 6.10x_2 = 47.0. \end{cases}$ Then $x_1 + x_2 =$

(a) 11.0

(b) 11.5

(c) 12.0

(d) 12.5

(e) 10.0

Avg:

83.3333

8. Consider the data points (1, 2.1), (2, 3.5), and (3, 8.4). If $y(x) = b_0 e^{b_1 x}$ is the exponential function that fits these data, then $y(2.5) =$

(a) 5.5894

(b) 5.3274

(c) 5.1081

(d) 4.9153

(e) 4.6391

Avg:

68.5185

9. If Runge-Kutta method of order four is used to approximate the solutions for the initial value problem:

$$y' = 1 + y/t, \quad 1 \leq t \leq 2, \quad y(1) = 2,$$

with $h = 0.25$. Then $y(1.25) \approx$

(a) 2.7789
(b) 2.7431
(c) 2.8103
(d) 2.8504
(e) 2.6925

Avg:

70.3704

10. If the matrix $\begin{bmatrix} 2 & 3 & -1 \\ 4 & 4 & -1 \\ -2 & -3 & 4 \end{bmatrix}$ is expressed in LU form, where $L = \begin{bmatrix} 1 & 0 & 0 \\ l_1 & 1 & 0 \\ l_2 & l_3 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$, then $l_1 + l_2 + l_3 + u_{11} + u_{22} + u_{33} =$

(a) 4
(b) 2
(c) 0
(d) -1
(e) -2

Avg:

70.3704

11. If the following system is solved using Gauss-Seidel iterative method,

$$\begin{cases} 10x_1 - x_2 = 9, \\ -x_1 + 10x_2 - 2x_3 = 7, \\ -2x_2 + 10x_3 = 6, \end{cases}$$

with $\mathbf{x}^{(0)} = (0, 0, 0)^t$. Then $\mathbf{x}^{(2)} =$

Avg:

74.0741

(a) $(0.979, 0.9495, 0.7899)^t$
(b) $(0.975, 0.9294, 0.7901)^t$
(c) $(0.971, 0.9605, 0.7795)^t$
(d) $(0.977, 0.9387, 0.7978)^t$
(e) $(0.973, 0.9760, 0.7880)^t$

12. If the system $\begin{cases} 4x_1 + 3x_2 = 24, \\ 3x_1 + 4x_2 = 30, \end{cases}$ is solved using Conjugate Gradient method with $\mathbf{x}^{(\mathbf{0})} = (0, 0)^t$. Then $x_1^{(1)} + x_2^{(1)} =$

Avg:

83.3333

(a) 7.7958
(b) 7.7235
(c) 7.8010
(d) 7.8315
(e) 7.8745

13. If $P_2(x)$ is the least squares polynomial of degree 2 for the data points:

$$(1, 1.5), (2, 2.3), (3, 3.8), (4, 5.3), (5, 6.7).$$

Then $P_2(1) =$

Avg:

68.5185

(a) 1.4114
(b) 1.5000
(c) 1.4505
(d) 1.5535
(e) 1.6000

14. If $P_1(x) = a + 0.5x$ is the linear least squares polynomial for the data points:

$$(0, 1), (1, 0), (2, 2), (3, b), (4, 2).$$

Then $P_1(3.5) =$

Avg:

59.2593

(a) 2.35
(b) 2.45
(c) 2.60
(d) 1.95
(e) 1.75

15. If the singular value decomposition for $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \end{bmatrix} = USV^t$. Then $S * \begin{bmatrix} \sqrt{2} \\ \sqrt{2} \\ 0 \end{bmatrix} =$

(a) $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$

(b) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(d) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(e) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

Avg:

74.0741

16. If the singular value decomposition of

$$A = USV^t = \begin{bmatrix} -0.5257 & -0.8507 \\ -0.8507 & 0.5257 \end{bmatrix} \begin{bmatrix} 2.6180 & 0 \\ 0 & 0.3820 \end{bmatrix} \begin{bmatrix} -0.5257 & -0.8507 \\ -0.8507 & 0.5257 \end{bmatrix},$$

$\mathbf{b} = \begin{bmatrix} 3.5 \\ 8.2 \end{bmatrix}$, and $\mathbf{x} = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$. Then the least squares polynomial $P(x) = a_1 x + a_0$ is

(a) $4.70x - 1.20$

(b) $4.72x - 1.15$

(c) $4.74x - 1.25$

(d) $4.76x - 1.27$

(e) $4.80x - 1.30$

Avg:

79.6296

17. Let $\mathbf{x}^{(0)} = (0,0)^t$. If the first iteration of Newton's method for the nonlinear system $\begin{cases} x_1 - x_1^2 + 4x_2 - 12 = 0, \\ (x_1-2)^2 + (2x_2-3)^2 - 25 = 0, \end{cases}$ is $(x_1^{(1)}, x_2^{(1)})$, then $x_1^{(1)} + x_2^{(1)} =$

Avg:

70.3704

(a) -33
(b) -31
(c) -28
(d) -23
(e) 12.5

18. Let $\mathbf{x}^{(0)} = (1,1)^t$ and $\alpha = 0.02$. If the first iteration of the Steepest Descent method for the nonlinear system $\begin{cases} 3x_1^2 - x_2^2 = 0, \\ 3x_1x_2^2 - x_1^3 - 1 = 0, \end{cases}$ is $(x_1^{(1)}, x_2^{(1)})$, then $x_1^{(1)} + x_2^{(1)} =$

Avg:

75.9259

(a) 1.44
(b) 1.47
(c) 1.50
(d) 1.53
(e) 1.39

19. If the Linear Finite Difference method is used to approximate the solution of the boundary value problem

$$y'' - y = 0, \quad 0 \le x \le 1, \quad y(0) = 1, \quad y(1) = e,$$

with $h = 1/3$, then $y(2/3) \approx$

(a) 1.9494
(b) 1.9463
(c) 1.4860
(d) 1.9409
(e) 1.7633

Avg:

81.4815

20. If the Linear Finite Difference method is used to approximate the solution of the boundary value problem

$$y'' + (x+1)y' - 2y = 1 - x, \quad 0 \le x \le 1, \quad y(0) = -1, \; y(1) = 0,$$

with $h = 0.25$. Given that the resulting system of equations is of the form $Aw = b$, then the sum of all elements in the second row of A equals to

Avg:

70.3704

(a) $1/8$
(b) $3/8$
(c) $2/7$
(d) $4/7$
(e) $-3/7$