King Fahd University of Petroleum and Minerals
Information Technology Center
Computing Services Section

Dept. of: Math
Course: math371 First Major Exam
Semester: 242

Thursday, February 20, 2025

			Raw Score	% Score
Total no. of Students:	30	**Course Mean:**	43.33	61.91
Course Std. Dev. :	17.49	**Max. Score:**	70	100
		Min. Score:	5	7.14

Average (%) of each question

1.0000	46.6667
2.0000	93.3333
3.0000	76.6667
4.0000	66.6667
5.0000	53.3333
6.0000	60.0000
7.0000	40.0000
8.0000	40.0000
9.0000	56.6667
10.0000	63.3333
11.0000	76.6667
12.0000	63.3333
13.0000	70.0000
14.0000	53.3333

Question with highest average
Q2

Question with lowest average
Q7 and Q8

Master

King Fahd University of Petroleum and Minerals
Department of Mathematics - College of Computing and Mathematics
Math 371- Numerical computing
Exam I, Term 242,
Date: 19-03-2025
Net Time Allowed: 90 minutes

Master

Q1, 1.1 (Similar to Q10)

Let $P_2(x)$ be the second Taylor polynomial of $f(x) = \sqrt{x+1}$ about $x_0 = 0$. The absolute error in using $P_2(x)$ to approximate $\sqrt{1.25}$ is

a. $\mathbf{0.0009}$
b. 0.3117
c. 0.0703
d. 0.0005
e. 0.0010

no students = 30
Average = 46.6667 %

Q2 1.2 (Similar to Q7d)

Given

$$f(x) = \frac{\sqrt{x} + \sqrt{x-2}}{\sqrt{x} - \sqrt{x-2}}.$$

Use three-digit rounding arithmetic to compute $f(13)$

a. $\mathbf{23.9}$
b. 23.8
c. 23.1
d. 24.0
e. 23.0

no students = 30
Average = 93.3333 %

Q3 2.1 (Similar to Q10b)

The first iteration of the bisection method that guarantees the approximation of the solution of

$$x^2 - 1 = e^{1-x^2}$$

on $[-2,0]$ within $10^{-0.75}$ accuracy is:

a. $\mathbf{-1.3750}$
b. -1.5000
c. -0.2500
d. -2.6250
e. -2.8750

no students = 30
Average = 76.6667 %

Master

Q4 2.2 (Similar to Q13a)

Use the fixed-point iteration with $p_0 = 1.75$ to find the third iterate, p_3, in approximating the solution of the equation

$$x = e^{-x} + 1$$

no students = 30
Average = 66.6667 %

- **a. $\mathbf{1.2700}$**
- b. 1.1737
- c. 1.3213
- d. -2.3206
- e. -0.5762

Q5 2.3 (Similar to Q11)
The first iteration of the Newton method that guarantees the approximation of the solution of

$$e^x = 3\cos x + 2x$$

within 10^{-3} accuracy taking $p_0 = 1$, is:

no students = 30
Average = 53.3333 %

- **a. $\mathbf{1.2397}$**
- b. 1.2405
- c. 1.2784
- d. 2.2951
- e. 1.7056

Q6 3.1 (Similar to Q10)
Let $f(x) = \sqrt{x - x^2}$, $0 \le x \le 1$, and $P_2(x)$ be the second Lagrange polynomial interpolating f at $x_0 = 0$, x_1, and $x_2 = 1$. The largest values of x_1 for which $f(0.75) - P_2(0.75) = 0.01$ is

- **a. $\mathbf{0.7313}$**
- b. 1.0000
- c. 0.5000
- d. 0.2500
- e. 0.8750

no students = 30
Average = 60.0000 %

Q7 3.1 (Similar to Q13)
Let $P_2(x)$ be the second Lagrange polynomial interpolating $f(x)$ at the points $(1,0)$, $(1.1, 0.0953)$, and $(1.3, 0.2624)$. Suppose $f'(x) = \frac{1}{x}$, find the smallest bound on the absolute error $|f(1.2) - P_2(1.2)|$ is

- **a. $\mathbf{6.77 \times 10^{-4}}$**
- b. 4.0×10^{-3}
- c. 4.27×10^{-1}
- d. 6.49×10^{-2}
- e. 5.0×10^{-4}

no students = 30
Average = 40.0000%

Q8 3.5 (Similar to Q1)

Construct a natural cubic spline that interpolates a function $f(x)$ at the points $(1,0)$, $(2,1)$ and $(4,2)$. Using the spline to approximate $f(3)$ gives

a. $\mathbf{1.6250}$
b. 1.2500
c. 1.2188
d. 1.3125
e. 1.5000

no students = 30
Average = 40.0000%

Q9 3.5 (Similar to Q14)

A clamped cubic spline $S(x)$ for a function f is defined on $[0,2]$ by

$$S(x) = \begin{cases} S_0(x) = 1 + Bx + x^2 - 2x^3, & 0 \le x \le 1, \\ S_1(x) = 1 + b(x-1) + c(x-1)^2 + 7(x-1)^3, & 1 \le x \le 2. \end{cases}$$

Then,

$f'(0) + f'(2) =$

a. $\mathbf{9}$
b. 8
c. -3
d. -5
e. 1

no students = 30
Average = 56.6667%

Q10 4.1 (Similar to Q9b)

Consider the data below:

x	$f(x)$	$f'(x)$
-2.6	7.1804	ℓ
-2.4	6.2093	m
-2.2	5.3203	n

Suppose the first derivative $f'(x)$ is computed using the most accurate three-point formulas possible, then

$\ell + m + n =$

a. $\mathbf{-13.9508}$
b. -5.0608
c. -4.6503
d. -4.2398
e. -4.8555

no students = 30
Average = 63.3333 %

Q11 4.1 (Similar to Q20)

Consider the data below:

x	1.10	1.20	1.30	1.40	1.50
y	11.59006	13.78176	14.04276	14.30741	16.86187

Using a step-size $h = 0.1$, $f''(1.3) \approx$

no students = 30
Average = 76.6667 %

a. **0.3650**
b. 0.0365
c. 0.1825
d. 0.0183
e. 0.0122

Q12 4.4 (Similar to Q13)

The minimum value of n required to approximate

$$\int_0^3 \frac{1}{x+4}dx$$

to within 10^{-5} accuracy by the composite Trapezoidal rule is

no students = 30
Average = 63.3333%

a. **84**
b. 83
c. 100
d. 89
e. 7

Q13 1.2 (Similar to Q9 and Q10)

Suppose p^* is used to approximate a number p, with relative error at most 10^{-3}. Find the largest interval in which p^* must lie?

no students = 30
Average = 70.0000%

a. **$p - 10^{-3}|p| \le p^* \le p + 10^{-3}|p|$**
b. $p - 10^{-3}p \le p^* \le p + 10^{-3}p$
c. $p - 10^{-3} \le p^* \le p + 10^{-3}$
d. $p + 10^{-3} \le p^* \le p - 10^{-3}$
e. $-p - 10^{-3}|p| \le p^* \le -p + 10^{-3}|p|$

Q14 4.4 (Similar to Q9 and Q10)

Suppose that $f(0) = 1$, $f(0.75) = 2.5$, $f(1) = 2$, $f(0.25) = \alpha$, and $f(0.5) = \beta$. Assume the composite Trapezoidal rule gives 2 and the composite Simpson's rule gives 3 for approximating $\int_0^1 f(x)dx$. Then, $\alpha - \beta =$

a. **11**
b. -11
c. $-\frac{7}{2}$
d. $\frac{15}{2}$
e. $\frac{5}{2}$

no students = 30
Average = 53.3333%