King Fahd University of Petroleum and Minerals
Information Technology Center
Computing Services Section

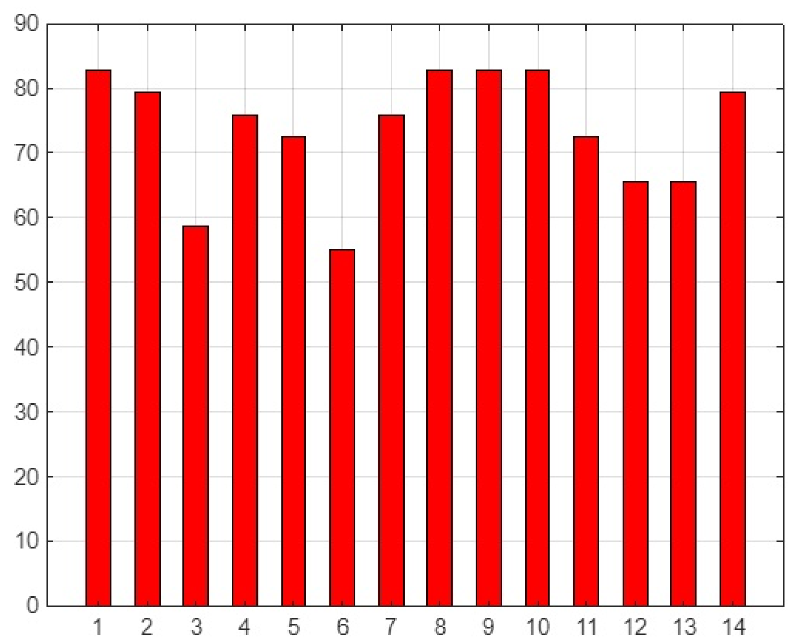
Dept. of: Math
Course: math371 Second Maior Exam
Semseter: 242

Tuesday, April 15, 2025

			Raw Score	% Score
Total no. of Students:	29	**Course Mean:**	51.55	73.65
Course Std. Dev. :	18.62	**Max. Score:**	70	100
		Min. Score:	15	21.43

Average (%) of each question

Q1	82.7586
Q2	79.3103
Q3	58.6207
Q4	75.8621
Q5	72.4138
Q6	55.1724
Q7	75.8621
Q8	82.7586
Q9	82.7586
Q10	82.7586
Q11	72.4138
Q12	65.5172
Q13	65.5172
Q14	79.3103



Question with highest average
Q1 Q8 Q9 Q10

Question with lowest average
Q3

1. Which of the following is always true about the differential equation

$$y' = f(t, y), \quad a \le t \le b, \quad y(a) = \alpha$$

on the domain $D = \{(t, y) : a \le t \le b, -\infty < y < \infty\}$

(a) If f is continuous on D and Lipschitz in the variable y on D, then the problem has a unique solution.
(b) If f is continuous on D, then the problem has a unique solution.
(c) If f is continuous on D and Lipschitz in the variable y on D, the problem has a solution but this solution is not unique.
(d) If f is continuous on D, then the problem is well-posed.
(e) If f is continuous on D and Lipschitz in the variable y on D, the problem is not necessarily

no students = 29
Average = 82.7586 %

2. Approximating the solution of the initial value problem

$$y' = te^{3t} - 2y, \quad 0 \le t \le 1, \quad y(0) = 1$$

by Euler's method with $h = 0.25$ gives $y(0.5) \approx$

(a) 0.3823
(b) 0.5000
(c) 0.7513
(d) 0.1323
(e) 0.9398

no students = 29
Average = 79.3103 %

3. Consider the initial value problem

$$y' - 1 = \frac{y}{t}, \quad 1 \le t \le 2, \quad y(1) = 2,$$

with exact solution $y(t) = t\ln(t) + 2t$. Use the Euler error estimate with $h = 0.2$ to find the smallest bound on the error $|y(1.8) - w_4|$

(a) 1.226×10^{-1}
(b) 6.128×10^{-2}
(c) 2.451×10^{-1}
(d) 5.050×10^{-1}
(e) 2.152×10^{-2}

no students = 29
Average = 58.6207 %

4. Consider the initial value problem

$$y' + 2y = e^{-t}, \quad 0 \le t \le 2, \quad y(0) = 1,$$

with exact solution $y(t) = e^{-t}$. Using second-order Runge-Kutta method with $N = 10$, the absolute error in approximating $y(0.4)$ is

(a) 3.352×10^{-3}
(b) 2.237×10^{-3}
(c) 7.033×10^{-3}
(d) 6.778×10^{-2}
(e) 2.855×10^{-4}

no students = 29
Average = 75.8621 %

5. Approximating the solution of the initial value problem

$$y' = e^{t-y}, \quad 0 \le t \le 0.9, \quad y(0) = 1$$

by the fourth-order Runge-Kutta (RK4) method with $N = 3$ gives $y(0.3) \approx$

(a) 1.1211

(b) 0.3000

(c) 1.2642

(d) 1.4298

(e) 2.0463

no students = 29
Average = 72.4138 %

6. The value of α so that the linear system

$$\begin{aligned} x_1 - x_2 + \alpha x_3 &= 3 \\ -x_1 + 2x_2 - \alpha x_3 &= -2 \\ \alpha x_1 + x_2 + 4x_3 &= 9 \end{aligned}$$

has no solution is

(a) $\alpha = -2$

(b) $\alpha = 1$

(c) $\alpha = -4$

(d) $\alpha = 4$

(e) $\alpha = 2$

no students = 29
Average = 55.1724 %

7. Consider the linear system

$$\begin{aligned} -6x_1 - 12x_2 - x_3 &= -9 \\ 2x_1 + x_2 - x_3 &= 8 \\ 5x_1 + 12x_2 + x_3 &= 7 \end{aligned}$$

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Average = 75.8621%

Let $R_i^{(k)}$ denote the i-th row of the augmented matrix at stage k of the Gaussian elimination and $R_i^{(k)} \leftrightarrow R_j^{(k)}$ be an interchange of the i-th and j-th rows.

The row interchanges required to solve the system by partial pivoting are

(a) No row interchanges required in stages 1 and 2

(b) Stage 1: $R_1^{(1)} \leftrightarrow R_3^{(1)}$, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$

(c) Stage 1: No row interchanges, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$

(d) Stage 1: $R_1^{(1)} \leftrightarrow R_3^{(1)}$, Stage 2: No row interchanges

(e) Stage 1: $R_1^{(1)} \leftrightarrow R_2^{(1)}$, Stage 2: $R_2^{(2)} \leftrightarrow R_3^{(2)}$

8. Consider the matrix

$$A = \begin{bmatrix} 0 & 0 & -1 & 1 \\ 1 & 1 & -1 & 2 \\ -1 & -1 & 2 & 0 \\ 1 & 2 & 0 & 2 \end{bmatrix}$$

The permutation matrix P such that

$$PA = \begin{bmatrix} 1 & 1 & -1 & 2 \\ 1 & 2 & 0 & 2 \\ -1 & -1 & 2 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

is

no students = 29
Average = 70.3704 %

(a) $P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

(b) $P = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

(c) $P = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

(d) $P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(e) $P = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

9. Consider the linear system $Ax = b$, given by

$$A = \begin{bmatrix} 2 & 1 & -1 \\ -4 & 2 & 4 \\ 6 & 3 & 2 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix}$$

Let $A = LU$ be an LU-factorization of the matrix A, where L is lower triangular of the form

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_1 & 1 & 0 \\ l_2 & l_3 & 1 \end{bmatrix}$$

and U is a corresponding upper triangular matrix. Then $l_1 - l_2 - l_3 =$

(a) -5
(b) -1
(c) 1
(d) 0
(e) 3

no students = 29
Average = 82.7586 %

10. The ℓ_∞ norm of the matrix

$$A = \begin{bmatrix} 4 & -1 & 7 \\ -1 & 4 & 0 \\ -7 & 0 & 4 \end{bmatrix}$$

is

(a) 12
(b) 5
(c) 11
(d) 10
(e) 15

no students = 29
Average = 82.7586 %

11. Given the matrix

$$A = \begin{bmatrix} 2 & 2 \\ -1 & 2 \end{bmatrix},$$

$\|A\|_2 =$

no students = 29
Average = 72.4138 %

(a) 3
(b) 9
(c) 1
(d) 2
(e) 4

12. Let $x^{(1)}$ and $x^{(2)}$ be the first and second iterates, respectively, of the Jacobi iteration with $x^{(0)} = (1, 0, -1)^t$ in approximating the solution of the system

$$\begin{aligned} 3x_1 - x_2 + x_3 &= 3 \\ 3x_1 + 6x_2 + 2x_3 &= 7 \\ 3x_1 + 3x_2 + 7x_3 &= 1 \end{aligned}$$

no students = 29
Average = 65.5172 %

Then $\|x^{(2)} - x^{(1)}\|_\infty =$

(a) $\frac{4}{7}$
(b) $\frac{17}{42}$
(c) $\frac{2}{21}$
(d) $\frac{11}{14}$
(e) $\frac{4}{3}$

13. Consider the system

$$\begin{aligned} 10x_1 + 5x_2 &= 2 \\ 5x_1 + 10x_2 - 4x_3 &= 0 \\ -4x_2 + 8x_3 - x_4 &= 2 \\ -x_3 + 5x_4 &= 5 \end{aligned}$$

no students = 29
Average = 65.5172 %

The first iterate of the Gauss-Seidel method in solving the system with initial guess $x^{(0)} = (1, 0, -1, 1)^t$ is

(a) $x^{(1)} = (0.2, -0.5, 0.125, 1.025)^t$
(b) $x^{(1)} = (0.2, -0.5, 0.375, 0.8)^t$
(c) $x^{(1)} = (0.2, -0.9, -0.075, 0.985)^t$
(d) $x^{(1)} = (0.2, -0.9, 0.375, 0.925)^t$
(e) $x^{(1)} = (0.2, -0.5, 0.125, 0.8)^t$

14. Consider the linear system

$$\begin{aligned} 6x_1 + 3x_2 &= 4 \\ 2x_1 - 3x_2 &= 1 \end{aligned}$$

Let $x^{(1)}$ be the first iteration of the conjugate gradient method, using the initial guess $x^{(0)} = (1, -1)^t$ and the search direction $v_1 = (1, 0)^t$. Then, $\|x^{(1)}\|_2 =$

(a) 1.5366
(b) 1.3611
(c) 2.2669
(d) 0.1667
(e) 1.0000

no students = 29
Average = 79.3103 %

Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	C 10	C 6	C 3	A 8
2	A	C 5	B 1	D 11	B 13
3	A	A 7	B 8	C 9	E 4
4	A	C 2	B 9	B 4	B 1
5	A	E 11	E 12	A 7	B 10
6	A	A 3	D 10	B 13	C 3
7	A	A 6	E 2	E 5	E 5
8	A	D 14	C 11	A 2	B 11
9	A	A 1	C 14	C 14	C 6
10	A	D 9	B 3	B 8	D 9
11	A	D 8	C 4	A 1	D 2
12	A	B 13	D 13	D 10	A 12
13	A	D 4	B 7	C 12	C 14
14	A	D 12	A 5	A 6	C 7

Answer Counts

V	A	B	C	D	E
1	4	1	3	5	1
2	1	5	4	2	2
3	4	3	4	2	1
4	2	4	4	2	2