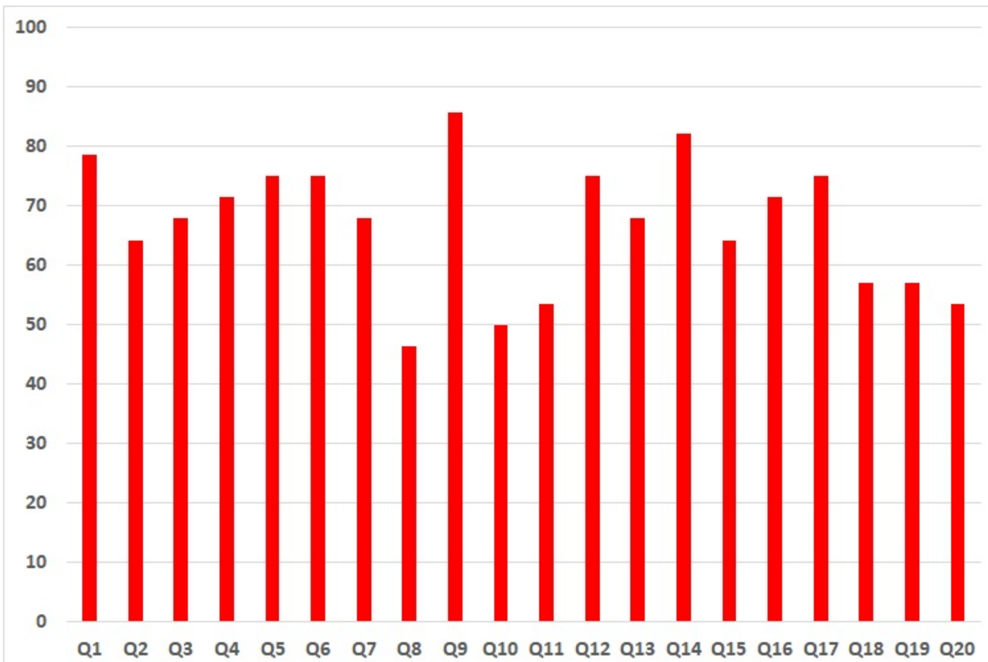
King Fahd University of Petroleum and Minerals
Information Technology Center
Computing Services Section

Dept. of: Math
Course: MATH371 Final Exam
Semseter: 242

Monday, May 26, 2025

			Raw Score	% Score
Total no. of Students	28	**Course Mean:**	66.96	66.96
Course Std. Dev. :	21.96	**Max. Score:**	100	100
		Min. Score:	20	20

Q	Average
Q1	78.5714
Q2	64.2857
Q3	67.8571
Q4	71.4286
Q5	75
Q6	75
Q7	67.8571
Q8	46.4286
Q9	85.7143
Q10	50
Q11	53.5714
Q12	75
Q13	67.8571
Q14	82.1429
Q15	64.2857
Q16	71.4286
Q17	75
Q18	57.1429
Q19	57.1429
Q20	53.5714



King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 371

Final Exam

242

May 25, 2025

Net Time Allowed: 120 Minutes

MASTER VERSION

1. Let $P_2(x)$ be the second Taylor polynomial for $f(x) = \ln(x+1)$ about $x_0 = 0$. Then the smallest upper bound, by the Taylor theorem, for $|f(0.5) - P_2(0.5)|$ is

   (a) 4.17×10^{-2} ______ (correct)
   (b) 1.25×10^{-1}
   (c) 3.33×10^{-1}
   (d) 2.08×10^{-2}
   (e) 8.34×10^{-2}

Average

78.5714

2. Consider $f(x) = x - 10\pi + 6e$. Using three-digit rounding arithmetic, $f(-3/62) \approx$

   (a) -15.1 ______ (correct)
   (b) -15.2
   (c) 15.0
   (d) 14.9
   (e) 14.8

Average

64.2857

3. Consider the problem

$$e^x - x^2 + 3x - 2 = 0, \quad -1 \le x \le 2.$$

The minimum number of iteration required by the bisection method to guarantee 10^{-8} accuracy is

(a) 29 ________________________________ (correct)

(b) 27

(c) 35

(d) 25

(e) 15

Average

67.8571

4. Using the secant method with $p_0 = -1$ and $p_1 = 0$ to approximate the solution of

$$-x^3 = \cos(x), \quad \text{we get} \quad p_3 =$$

(a) -1.2521 ________________________________ (correct)

(b) -0.7523

(c) -0.8803

(d) -0.8657

(e) -0.8655

Average

71.4286

5. Let $P_2(x)$ be the interpolating polynomial for the data $(0, y), (1, 3), (2, 2)$. Suppose the coefficient of x^2 in $P_2(x)$ is 6, then $y =$

   (a) 16 ______________________________ (correct)
   (b) 10
   (c) 8
   (d) 2.5
   (e) 4.5

Average

75

6. Using the composite trapezoidal rule with $n = 4$,

$$\int_0^2 \frac{1}{x+4} dx \approx$$

   (a) 0.4062 ______________________________ (correct)
   (b) 0.8124
   (c) 0.2031
   (d) 0.4055
   (e) 0.3914

Average

75

7. Consider the initial value problem

$$y' = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5.$$

Using the Euler method with $n = 4$, $y(1.0) \approx$

(a) 2.2500 ________________________________ (correct)

(b) 3.3750

(c) 4.4375

(d) 1.1250

(e) 1.6875

Average

67.8571

8. Consider the matrix A and the vector $\mathbf{b}$: $A = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 4 & -1 \\ -2 & -3 & 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}.$

Let $A = LU$ be a factorization of A, where L is lower triangle matrix with 1 on its diagonal and U is upper triangle. The solution of system $L\mathbf{y} = \mathbf{b}$, where $\mathbf{y} = (y_1, y_2, y_3)^t$ is

(a) $\mathbf{y} = (2, -5, 3)^t$ ________________________________ (correct)

(b) $\mathbf{y} = (-3, 3, 1)^t$

(c) $\mathbf{y} = (0.1667, 0.667, 0.3333)^t$

(d) $\mathbf{y} = (-2, 5, -3)^t$

(e) $\mathbf{y} = (2, -5, -1)^t$

Average

46.4286

9. The first iteration of the Gauss-Seidel method for solving the linear system

$$
\begin{aligned}
2x_1 - x_2 &= 4 \\
-x_1 + 2x_2 - x_3 &= 9 \\
-x_2 + 2x_3 &= 6
\end{aligned}
$$

using the initial guess $\mathbf{x}^{(0)} = (0,0,0)^t$ gives

(a) $\mathbf{x}^{(1)} = (2, 5.5, 5.75)^t$ ______ (correct)

(b) $\mathbf{x}^{(1)} = (2, 4.5, 3)^t$

(c) $\mathbf{x}^{(1)} = (4, 9, 6)^t$

(d) $\mathbf{x}^{(1)} = (2, 5.5, 3)^t$

(e) $\mathbf{x}^{(1)} = (2, 5.5, 3)^t$

Average

85.7143

10. Consider the linear system

$$
\begin{aligned}
6x_1 + 3x_2 &= 4 \\
2x_1 - 3x_2 &= 1
\end{aligned}
$$

Let $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ be the first and second iteration, respectively, of the conjugate gradient method using the initial guess $\mathbf{x}^{(0)} = (1,-1)^t$ and the conjugate directions $\mathbf{v}^{(1)} = (1,0)^t$ and $\mathbf{v}^{(2)} = (-1,2)^t$. Then, $\|\mathbf{x}^{(1)} - \mathbf{x}^{(2)}\|_\infty =$

(a) 1.0833 ______ (correct)

(b) 1.1667

(c) 0.5417

(d) 1.000

(e) 0.6250

Average

50

11. Consider the following data

x	1.0	2.0	3.0
y	2.3551	ℓ	3.1116

.

Let $f(x) = 2.1021e^{ax}$ by a least square curve fitted to the data. Then, $a =$

(a) 0.1392 ______________________________ (correct)

(b) 1.1495

(c) -1.9711

(d) 0.3782

(e) -0.9723

Average

53.5714

12. Consider the following data

x	0	1.0	2.0
y	0.2849	0.8934	β

.

Let $P_2(x) = a_2x^2 + 0.5255x + a_0$ be a least square polynomial fitted to the data. Then, $P_2(1.75) =$

(a) 1.4587 ______________________________ (correct)

(b) 1.6679

(c) 1.8728

(d) 2.0124

(e) 2.2457

Average

75

13. Using the Gram-Schmidt process to find a set of orthogonal vectors u_1, u_2, u_3 from the linearly independent vectors

$$\mathbf{v}_1 = (2, -1, 1), \quad \mathbf{v}_2 = (1, 0, 1), \quad \mathbf{v}_3 = (0, 2, 0),$$

gives

(a) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, \frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (\frac{2}{3}, \frac{2}{3}, -\frac{2}{3})^t$ ________ (correct)

(b) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, -\frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (\frac{2}{3}, \frac{2}{3}, -\frac{2}{3})^t$

(c) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, -\frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (2, -\frac{1}{3}, 1)^t$

(d) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, \frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (2, -\frac{1}{3}, 1)^t$

(e) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, \frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (-\frac{2}{3}, \frac{2}{3}, -\frac{2}{3})^t$

Average

67.8571

14. The singular values of the matrix $\begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$ are

(a) $s_1 = 2.6762, \quad s_2 = 0.9153$ ________ (correct)

(b) $s_1 = 2.5959, \quad s_2 = 1.1231$

(c) $s_1 = 1.2644, \quad s_2 = 0.7380$

(d) $s_1 = 7.1623, \quad s_2 = 0.8377$

(e) $s_1 = 2.6762, \quad s_2 = 0.8377$

Average

82.1429

15. The matrix $A = \begin{bmatrix} 2 & 1 \\ -1 & 1 \\ 1 & 1 \\ 2 & -1 \end{bmatrix}$ has the singular value decomposition $A = USV^t$, with

Average

$$S = \begin{bmatrix} 3.1623 & 0 \\ 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad V = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

64.2857

The first two columns of the matrix U obtained using S and V are:

(a) $\mathbf{u}_1 = (-0.6325, 0.3162, -0.3162, -0.6325)^t$, $\mathbf{u}_2 = (-0.5, -0.5, -0.5, 0.5)^t$ (correct)

(b) $\mathbf{u}_1 = (-0.6325, 0.3162, -0.3162, -0.6325)^t$, $\mathbf{u}_2 = (0.5, 0.5, 0.5, 0.5)^t$

(c) $\mathbf{u}_1 = (0.6325, 0.3162, 0.3162, 0.6325)^t$, $\mathbf{u}_2 = (-0.5, -0.5, -0.5, 0.5)^t$

(d) $\mathbf{u}_1 = (0.6325, -0.3162, 0.3162, 0.6325)^t$, $\mathbf{u}_2 = (0.5, 0.5, 0.5, -0.5)^t$

(e) $\mathbf{u}_1 = (0.6325, 0.3162, 0.3162, 0.6325)^t$, $\mathbf{u}_2 = (0.5, 0.5, 0.5, 0.5)^t$

16. Let A be a real matrix with m rows and n column, $m \geq n$ and $A = USV^t$. Which of the following statements is not always true

(a) S is invertible ______ (correct)

(b) U and V are orthogonal

Average

(c) U and V are always invertible

(d) All entries of S are non-negative

71.4286

(e) U is $m \times m$ and V is $n \times n$

17. Let $\mathbf{x} = (x_1, x_2)^t$ and $\mathbf{x}^{(1)}$ be the first iterate of the Newton's method in solving

$$4x_1^2 - 20x_1 + \frac{1}{4}x_2^2 = -8$$
$$-\frac{1}{2}x_1x_2^2 + 2x_1 - 5x_2 = -8,$$

Average

75

with $\mathbf{x}^{(0)} = (0, 0)^t$. Then $\mathbf{x}^{(1)} =$

(a) $(0.4, 1.76)^t$ ____(correct)

(b) $(-0.4, 1.76)^t$

(c) $(0.4, 1.92)^t$

(d) $(1.6, 0.56)^t$

(e) $(0.4, -1.44)^t$

18. Let $\mathbf{x} = (x_1, x_2)^t$ and $\mathbf{x}^{(1)}$ be the first iterate of the steepest descent method in solving the system of equations

$$3x_1^2 - x_2^2 = 0$$
$$3x_1x_2^2 - x_1^3 = 1,$$

with $\mathbf{x}^{(0)} = (1, \frac{1}{2})^t$, and $\alpha = 0.01$. Then $\mathbf{x}^{(1)} =$

(a) $(0.6138, 0.63)^t$ ____(correct)

Average

57.1429

(b) $(0.6138, 0.52)^t$

(c) $(0.6589, 0.57)^t$

(d) $(0.6589, 0.63)^t$

(e) $(0.7038, 0.51)^t$

19. Consider the boundary value problem

$$y'' = 4(y - x), \qquad 0 \le x \le 1, \qquad y(0) = 0,\ y(1) = 2.$$

Applying the linear finite difference method with $h = \frac{1}{3}$

Average

(a) $y(\frac{1}{3}) = 0.5343,\ y(\frac{2}{3}) = 1.1580$ ________________ (correct)
(b) $y(\frac{1}{3}) = 0.3904,\ y(\frac{2}{3}) = 0.8060$
(c) $y(\frac{1}{3}) = 0.2696,\ y(\frac{2}{3}) = 0.8073$ 57.1429
(d) $y(\frac{1}{3}) = 1.0377,\ y(\frac{2}{3}) = 1.7623$
(e) $y(\frac{1}{3}) = 1.7797,\ y(\frac{2}{3}) = 2.6203$

20. The linear finite difference scheme applied to the problem

$$y'' - y' - 2y = \cos(x), \qquad 0 \le x \le \frac{\pi}{2}, \qquad y(0) = -0.3,\ y(\frac{\pi}{2}) = -0.1,$$

with $\frac{\pi}{8}$ results in the system $Aw = b$, where

Average

53.5714

(a) $A = \begin{bmatrix} 2.3084 & -0.8037 & 0 \\ -1.1963 & 2.3084 & -0.8037 \\ 0 & -1.1963 & 2.3084 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.5014 \\ -0.1090 \\ -0.1394 \end{bmatrix}$ ________________ (correct)

(b) $A = \begin{bmatrix} 2.3084 & -0.8037 & 0 \\ -1.1963 & 2.3084 & -0.8037 \\ 0 & -1.1963 & 2.3084 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.2228 \\ -0.1090 \\ -0.1786 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1.6916 & -1.1963 & 0 \\ -0.8037 & 1.6916 & -1.1963 \\ 0 & -0.8037 & 1.6916 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.3836 \\ -0.1090 \\ -0.1786 \end{bmatrix}$

(d) $A = \begin{bmatrix} 2.3084 & -0.8037 & 0 \\ -1.1963 & 2.3084 & -0.8037 \\ 0 & -1.1963 & 2.3084 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.3836 \\ -0.1090 \\ -0.1786 \end{bmatrix}$

(e) $A = \begin{bmatrix} 1.6916 & -1.1963 & 0 \\ -0.8037 & 1.6916 & -1.1963 \\ 0 & -0.8037 & 1.6916 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.5014 \\ -0.1090 \\ -0.1394 \end{bmatrix}$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE01 **CODE01**

Math 371
Final Exam
242
May 25, 2025
Net Time Allowed: 120 Minutes

Name			
ID		**Sec**	

Check that this exam has 20 questions.

Important Instructions:

1. All types of smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. Consider the boundary value problem

$$y'' = 4(y - x), \qquad 0 \leq x \leq 1, \qquad y(0) = 0,\ y(1) = 2.$$

Applying the linear finite difference method with $h = \frac{1}{3}$

(a) $y(\frac{1}{3}) = 1.7797,\ y(\frac{2}{3}) = 2.6203$
(b) $y(\frac{1}{3}) = 1.0377,\ y(\frac{2}{3}) = 1.7623$
(c) $y(\frac{1}{3}) = 0.2696,\ y(\frac{2}{3}) = 0.8073$
(d) $y(\frac{1}{3}) = 0.5343,\ y(\frac{2}{3}) = 1.1580$
(e) $y(\frac{1}{3}) = 0.3904,\ y(\frac{2}{3}) = 0.8060$

2. Consider $f(x) = x - 10\pi + 6e$. Using three-digit rounding arithmetic, $f(-3/62) \approx$

(a) -15.2
(b) -15.1
(c) 14.9
(d) 14.8
(e) 15.0

3. Consider the following data

x	1.0	2.0	3.0
y	2.3551	ℓ	3.1116

Let $f(x) = 2.1021e^{ax}$ by a least square curve fitted to the data. Then, $a =$

(a) -1.9711
(b) 0.3782
(c) 1.1495
(d) -0.9723
(e) 0.1392

4. Let $P_2(x)$ be the second Taylor polynomial for $f(x) = \ln(x+1)$ about $x_0 = 0$. Then the smallest upper bound, by the Taylor theorem, for $|f(0.5) - P_2(0.5)|$ is

(a) 8.34×10^{-2}
(b) 1.25×10^{-1}
(c) 2.08×10^{-2}
(d) 3.33×10^{-1}
(e) 4.17×10^{-2}

5. The first iteration of the Gauss-Seidel method for solving the linear system

$$\begin{aligned} 2x_1 - x_2 &= 4 \\ -x_1 + 2x_2 - x_3 &= 9 \\ -x_2 + 2x_3 &= 6 \end{aligned}$$

using the initial guess $\mathbf{x}^{(0)} = (0,0,0)^t$ gives

(a) $\mathbf{x}^{(1)} = (2, 4.5, 3)^t$
(b) $\mathbf{x}^{(1)} = (2, 5.5, 5.75)^t$
(c) $\mathbf{x}^{(1)} = (2, 5.5, 3)^t$
(d) $\mathbf{x}^{(1)} = (2, 5.5, 3)^t$
(e) $\mathbf{x}^{(1)} = (4, 9, 6)^t$

6. Let A be a real matrix with m rows and n column, $m \geq n$ and $A = USV^t$. Which of the following statements is not always true

(a) U and V are orthogonal
(b) U and V are always invertible
(c) U is $m \times m$ and V is $n \times n$
(d) All entries of S are non-negative
(e) S is invertible

7. Let $\mathbf{x} = (x_1, x_2)^t$ and $\mathbf{x}^{(1)}$ be the first iterate of the steepest descent method in solving the system of equations

$$\begin{aligned} 3x_1^2 - x_2^2 &= 0 \\ 3x_1x_2^2 - x_1^3 &= 1, \end{aligned}$$

with $\mathbf{x}^{(0)} = (1, \frac{1}{2})^t$, and $\alpha = 0.01$. Then $\mathbf{x}^{(1)} =$

(a) $(0.6589, 0.57)^t$
(b) $(0.6589, 0.63)^t$
(c) $(0.6138, 0.52)^t$
(d) $(0.6138, 0.63)^t$
(e) $(0.7038, 0.51)^t$

8. Using the secant method with $p_0 = -1$ and $p_1 = 0$ to approximate the solution of

$$-x^3 = \cos(x), \quad \text{we get} \quad p_3 =$$

(a) -0.7523
(b) -0.8657
(c) -1.2521
(d) -0.8803
(e) -0.8655

9. Let $P_2(x)$ be the interpolating polynomial for the data $(0, y), (1, 3), (2, 2)$. Suppose the coefficient of x^2 in $P_2(x)$ is 6, then $y =$

(a) 16
(b) 8
(c) 2.5
(d) 10
(e) 4.5

10. Let $\mathbf{x} = (x_1, x_2)^t$ and $\mathbf{x}^{(1)}$ be the first iterate of the Newton's method in solving

$$\begin{aligned} 4x_1^2 - 20x_1 + \frac{1}{4}x_2^2 &= -8 \\ -\frac{1}{2}x_1x_2^2 + 2x_1 - 5x_2 &= -8, \end{aligned}$$

with $\mathbf{x}^{(0)} = (0, 0)^t$. Then $\mathbf{x}^{(1)} =$

(a) $(0.4, 1.92)^t$
(b) $(0.4, -1.44)^t$
(c) $(0.4, 1.76)^t$
(d) $(1.6, 0.56)^t$
(e) $(-0.4, 1.76)^t$

11. The singular values of the matrix $\begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$ are

(a) $s_1 = 7.1623, \quad s_2 = 0.8377$
(b) $s_1 = 2.5959, \quad s_2 = 1.1231$
(c) $s_1 = 2.6762, \quad s_2 = 0.8377$
(d) $s_1 = 2.6762, \quad s_2 = 0.9153$
(e) $s_1 = 1.2644, \quad s_2 = 0.7380$

12. Consider the initial value problem

$$y' = y - t^2 + 1, \quad 0 \le t \le 2, \quad y(0) = 0.5.$$

Using the Euler method with $n = 4$, $y(1.0) \approx$

(a) 4.4375
(b) 1.1250
(c) 2.2500
(d) 1.6875
(e) 3.3750

13. The linear finite difference scheme applied to the problem

$$y'' - y' - 2y = \cos(x), \qquad 0 \le x \le \frac{\pi}{2}, \qquad y(0) = -0.3, \ \ y(\frac{\pi}{2}) = -0.1,$$

with $\frac{\pi}{8}$ results in the system $Aw = b$, where

(a) $A = \begin{bmatrix} 2.3084 & -0.8037 & 0 \\ -1.1963 & 2.3084 & -0.8037 \\ 0 & -1.1963 & 2.3084 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.3836 \\ -0.1090 \\ -0.1786 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1.6916 & -1.1963 & 0 \\ -0.8037 & 1.6916 & -1.1963 \\ 0 & -0.8037 & 1.6916 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.3836 \\ -0.1090 \\ -0.1786 \end{bmatrix}$

(c) $A = \begin{bmatrix} 2.3084 & -0.8037 & 0 \\ -1.1963 & 2.3084 & -0.8037 \\ 0 & -1.1963 & 2.3084 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.5014 \\ -0.1090 \\ -0.1394 \end{bmatrix}$

(d) $A = \begin{bmatrix} 2.3084 & -0.8037 & 0 \\ -1.1963 & 2.3084 & -0.8037 \\ 0 & -1.1963 & 2.3084 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.2228 \\ -0.1090 \\ -0.1786 \end{bmatrix}$

(e) $A = \begin{bmatrix} 1.6916 & -1.1963 & 0 \\ -0.8037 & 1.6916 & -1.1963 \\ 0 & -0.8037 & 1.6916 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.5014 \\ -0.1090 \\ -0.1394 \end{bmatrix}$

14. Consider the linear system

$$6x_1 + 3x_2 = 4$$
$$2x_1 - 3x_2 = 1$$

Let $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ be the first and second iteration, respectively, of the conjugate gradient method using the initial guess $\mathbf{x}^{(0)} = (1, -1)^t$ and the conjugate directions $\mathbf{v}^{(1)} = (1, 0)^t$ and $\mathbf{v}^{(2)} = (-1, 2)^t$. Then, $\|\mathbf{x}^{(1)} - \mathbf{x}^{(2)}\|_\infty =$

(a) 1.000
(b) 1.1667
(c) 0.5417
(d) 1.0833
(e) 0.6250

15. Consider the problem

$$e^x - x^2 + 3x - 2 = 0, \quad -1 \leq x \leq 2.$$

The minimum number of iteration required by the bisection method to guarantee 10^{-8} accuracy is

(a) 15
(b) 25
(c) 27
(d) 29
(e) 35

16. Consider the following data

x	0	1.0	2.0
y	0.2849	0.8934	β

Let $P_2(x) = a_2x^2 + 0.5255x + a_0$ be a least square polynomial fitted to the data. Then, $P_2(1.75) =$

(a) 2.2457
(b) 1.4587
(c) 1.6679
(d) 1.8728
(e) 2.0124

17. The matrix $A = \begin{bmatrix} 2 & 1 \\ -1 & 1 \\ 1 & 1 \\ 2 & -1 \end{bmatrix}$ has the singular value decomposition $A = USV^t$, with

$$S = \begin{bmatrix} 3.1623 & 0 \\ 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad V = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

The first two columns of the matrix U obtained using S and V are:

(a) $\mathbf{u}_1 = (0.6325, 0.3162, 0.3162, 0.6325)^t, \quad \mathbf{u}_2 = (-0.5, -0.5, -0.5, 0.5)^t$
(b) $\mathbf{u}_1 = (0.6325, -0.3162, 0.3162, 0.6325)^t, \quad \mathbf{u}_2 = (0.5, 0.5, 0.5, -0.5)^t$
(c) $\mathbf{u}_1 = (-0.6325, 0.3162, -0.3162, -0.6325)^t, \quad \mathbf{u}_2 = (0.5, 0.5, 0.5, 0.5)^t$
(d) $\mathbf{u}_1 = (-0.6325, 0.3162, -0.3162, -0.6325)^t, \quad \mathbf{u}_2 = (-0.5, -0.5, -0.5, 0.5)^t$
(e) $\mathbf{u}_1 = (0.6325, 0.3162, 0.3162, 0.6325)^t, \quad \mathbf{u}_2 = (0.5, 0.5, 0.5, 0.5)^t$

18. Using the composite trapezoidal rule with $n = 4$,

$$\int_0^2 \frac{1}{x+4} dx \approx$$

(a) 0.8124
(b) 0.4055
(c) 0.2031
(d) 0.4062
(e) 0.3914

19. Consider the matrix A and the vector $\mathbf{b}$: $A = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 4 & -1 \\ -2 & -3 & 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}.$

Let $A = LU$ be a factorization of A, where L is lower triangle matrix with 1 on its diagonal and U is upper triangle. The solution of system $L\mathbf{y} = \mathbf{b}$, where $\mathbf{y} = (y_1, y_2, y_3)^t$ is

(a) $\mathbf{y} = (0.1667, 0.667, 0.3333)^t$
(b) $\mathbf{y} = (2, -5, 3)^t$
(c) $\mathbf{y} = (2, -5, -1)^t$
(d) $\mathbf{y} = (-2, 5, -3)^t$
(e) $\mathbf{y} = (-3, 3, 1)^t$

20. Using the Gram-Schmidt process to find a set of orthogonal vectors u_1, u_2, u_3 from the linearly independent vectors

$$\mathbf{v}_1 = (2, -1, 1), \quad \mathbf{v}_2 = (1, 0, 1), \quad \mathbf{v}_3 = (0, 2, 0),$$

gives

(a) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, \frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (2, -\frac{1}{3}, 1)^t$
(b) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, \frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (-\frac{2}{3}, \frac{2}{3}, -\frac{2}{3})^t$
(c) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, -\frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (\frac{2}{3}, \frac{2}{3}, -\frac{2}{3})^t$
(d) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, \frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (\frac{2}{3}, \frac{2}{3}, -\frac{2}{3})^t$
(e) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, -\frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (2, -\frac{1}{3}, 1)^t$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE02 **CODE02**

Math 371
Final Exam
242
May 25, 2025
Net Time Allowed: 120 Minutes

Name	

ID		**Sec**	

Check that this exam has 20 questions.

Important Instructions:

1. All types of smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. Consider the following data

x	1.0	2.0	3.0
y	2.3551	ℓ	3.1116

.

Let $f(x) = 2.1021e^{ax}$ by a least square curve fitted to the data. Then, $a =$

(a) -0.9723
(b) 0.1392
(c) 1.1495
(d) 0.3782
(e) -1.9711

2. Consider the initial value problem

$$y' = y - t^2 + 1, \quad 0 \le t \le 2, \quad y(0) = 0.5.$$

Using the Euler method with $n = 4$, $y(1.0) \approx$

(a) 3.3750
(b) 4.4375
(c) 2.2500
(d) 1.6875
(e) 1.1250

3. The first iteration of the Gauss-Seidel method for solving the linear system

$$\begin{aligned} 2x_1 - x_2 &= 4 \\ -x_1 + 2x_2 - x_3 &= 9 \\ -x_2 + 2x_3 &= 6 \end{aligned}$$

using the initial guess $\mathbf{x}^{(0)} = (0,0,0)^t$ gives

(a) $\mathbf{x}^{(1)} = (4,9,6)^t$
(b) $\mathbf{x}^{(1)} = (2,5.5,3)^t$
(c) $\mathbf{x}^{(1)} = (2,5.5,3)^t$
(d) $\mathbf{x}^{(1)} = (2,5.5,5.75)^t$
(e) $\mathbf{x}^{(1)} = (2,4.5,3)^t$

4. Using the secant method with $p_0 = -1$ and $p_1 = 0$ to approximate the solution of

$$-x^3 = \cos(x), \quad \text{we get} \quad p_3 =$$

(a) -0.8657
(b) -1.2521
(c) -0.8803
(d) -0.8655
(e) -0.7523

5. Using the composite trapezoidal rule with $n = 4$,

$$\int_0^2 \frac{1}{x+4} dx \approx$$

(a) 0.2031
(b) 0.4062
(c) 0.4055
(d) 0.3914
(e) 0.8124

6. Consider the problem

$$e^x - x^2 + 3x - 2 = 0, \quad -1 \le x \le 2.$$

The minimum number of iteration required by the bisection method to guarantee 10^{-8} accuracy is

(a) 27
(b) 15
(c) 35
(d) 29
(e) 25

7. Consider the linear system

$$6x_1 + 3x_2 = 4$$
$$2x_1 - 3x_2 = 1$$

Let $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ be the first and second iteration, respectively, of the conjugate gradient method using the initial guess $\mathbf{x}^{(0)} = (1, -1)^t$ and the conjugate directions $\mathbf{v}^{(1)} = (1, 0)^t$ and $\mathbf{v}^{(2)} = (-1, 2)^t$. Then, $\|\mathbf{x}^{(1)} - \mathbf{x}^{(2)}\|_\infty =$

(a) 0.6250
(b) 1.000
(c) 1.1667
(d) 1.0833
(e) 0.5417

8. Consider the matrix A and the vector $\mathbf{b}$: $A = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 4 & -1 \\ -2 & -3 & 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}.$

Let $A = LU$ be a factorization of A, where L is lower triangle matrix with 1 on its diagonal and U is upper triangle. The solution of system $L\mathbf{y} = \mathbf{b}$, where $\mathbf{y} = (y_1, y_2, y_3)^t$ is

(a) $\mathbf{y} = (0.1667, 0.667, 0.3333)^t$
(b) $\mathbf{y} = (2, -5, -1)^t$
(c) $\mathbf{y} = (-2, 5, -3)^t$
(d) $\mathbf{y} = (-3, 3, 1)^t$
(e) $\mathbf{y} = (2, -5, 3)^t$

9. Consider the boundary value problem

$$y'' = 4(y - x), \qquad 0 \le x \le 1, \qquad y(0) = 0,\ y(1) = 2.$$

Applying the linear finite difference method with $h = \frac{1}{3}$

(a) $y(\frac{1}{3}) = 0.3904,\ y(\frac{2}{3}) = 0.8060$
(b) $y(\frac{1}{3}) = 0.2696,\ y(\frac{2}{3}) = 0.8073$
(c) $y(\frac{1}{3}) = 1.7797,\ y(\frac{2}{3}) = 2.6203$
(d) $y(\frac{1}{3}) = 0.5343,\ y(\frac{2}{3}) = 1.1580$
(e) $y(\frac{1}{3}) = 1.0377,\ y(\frac{2}{3}) = 1.7623$

10. The linear finite difference scheme applied to the problem

$$y'' - y' - 2y = \cos(x), \qquad 0 \le x \le \frac{\pi}{2}, \qquad y(0) = -0.3,\ y(\frac{\pi}{2}) = -0.1,$$

with $\frac{\pi}{8}$ results in the system $Aw = b$, where

(a) $A = \begin{bmatrix} 1.6916 & -1.1963 & 0 \\ -0.8037 & 1.6916 & -1.1963 \\ 0 & -0.8037 & 1.6916 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.5014 \\ -0.1090 \\ -0.1394 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1.6916 & -1.1963 & 0 \\ -0.8037 & 1.6916 & -1.1963 \\ 0 & -0.8037 & 1.6916 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.3836 \\ -0.1090 \\ -0.1786 \end{bmatrix}$

(c) $A = \begin{bmatrix} 2.3084 & -0.8037 & 0 \\ -1.1963 & 2.3084 & -0.8037 \\ 0 & -1.1963 & 2.3084 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.2228 \\ -0.1090 \\ -0.1786 \end{bmatrix}$

(d) $A = \begin{bmatrix} 2.3084 & -0.8037 & 0 \\ -1.1963 & 2.3084 & -0.8037 \\ 0 & -1.1963 & 2.3084 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.3836 \\ -0.1090 \\ -0.1786 \end{bmatrix}$

(e) $A = \begin{bmatrix} 2.3084 & -0.8037 & 0 \\ -1.1963 & 2.3084 & -0.8037 \\ 0 & -1.1963 & 2.3084 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.5014 \\ -0.1090 \\ -0.1394 \end{bmatrix}$

11. Using the Gram-Schmidt process to find a set of orthogonal vectors u_1, u_2, u_3 from the linearly independent vectors

$$\mathbf{v}_1 = (2, -1, 1), \quad \mathbf{v}_2 = (1, 0, 1), \quad \mathbf{v}_3 = (0, 2, 0),$$

gives

(a) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, \frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (-\frac{2}{3}, \frac{2}{3}, -\frac{2}{3})^t$
(b) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, \frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (\frac{2}{3}, \frac{2}{3}, -\frac{2}{3})^t$
(c) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, \frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (2, -\frac{1}{3}, 1)^t$
(d) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, -\frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (2, -\frac{1}{3}, 1)^t$
(e) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, -\frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (\frac{2}{3}, \frac{2}{3}, -\frac{2}{3})^t$

12. Let A be a real matrix with m rows and n column, $m \geq n$ and $A = USV^t$. Which of the following statements is not always true

(a) S is invertible
(b) U is $m \times m$ and V is $n \times n$
(c) All entries of S are non-negative
(d) U and V are orthogonal
(e) U and V are always invertible

13. Let $P_2(x)$ be the second Taylor polynomial for $f(x) = \ln(x+1)$ about $x_0 = 0$. Then the smallest upper bound, by the Taylor theorem, for $|f(0.5) - P_2(0.5)|$ is

(a) 2.08×10^{-2}
(b) 8.34×10^{-2}
(c) 4.17×10^{-2}
(d) 3.33×10^{-1}
(e) 1.25×10^{-1}

14. The singular values of the matrix $\begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$ are

(a) $s_1 = 7.1623, \quad s_2 = 0.8377$
(b) $s_1 = 2.6762, \quad s_2 = 0.9153$
(c) $s_1 = 1.2644, \quad s_2 = 0.7380$
(d) $s_1 = 2.6762, \quad s_2 = 0.8377$
(e) $s_1 = 2.5959, \quad s_2 = 1.1231$

15. The matrix $A = \begin{bmatrix} 2 & 1 \\ -1 & 1 \\ 1 & 1 \\ 2 & -1 \end{bmatrix}$ has the singular value decomposition $A = USV^t$, with

$$S = \begin{bmatrix} 3.1623 & 0 \\ 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad V = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

The first two columns of the matrix U obtained using S and V are:

(a) $\mathbf{u}_1 = (0.6325, -0.3162, 0.3162, 0.6325)^t, \quad \mathbf{u}_2 = (0.5, 0.5, 0.5, -0.5)^t$
(b) $\mathbf{u}_1 = (-0.6325, 0.3162, -0.3162, -0.6325)^t, \quad \mathbf{u}_2 = (-0.5, -0.5, -0.5, 0.5)^t$
(c) $\mathbf{u}_1 = (0.6325, 0.3162, 0.3162, 0.6325)^t, \quad \mathbf{u}_2 = (0.5, 0.5, 0.5, 0.5)^t$
(d) $\mathbf{u}_1 = (-0.6325, 0.3162, -0.3162, -0.6325)^t, \quad \mathbf{u}_2 = (0.5, 0.5, 0.5, 0.5)^t$
(e) $\mathbf{u}_1 = (0.6325, 0.3162, 0.3162, 0.6325)^t, \quad \mathbf{u}_2 = (-0.5, -0.5, -0.5, 0.5)^t$

16. Let $P_2(x)$ be the interpolating polynomial for the data $(0, y), (1, 3), (2, 2)$. Suppose the coefficient of x^2 in $P_2(x)$ is 6, then $y =$

(a) 10
(b) 4.5
(c) 16
(d) 2.5
(e) 8

17. Let $\mathbf{x} = (x_1, x_2)^t$ and $\mathbf{x}^{(1)}$ be the first iterate of the Newton's method in solving

$$4x_1^2 - 20x_1 + \frac{1}{4}x_2^2 = -8$$
$$-\frac{1}{2}x_1x_2^2 + 2x_1 - 5x_2 = -8,$$

with $\mathbf{x}^{(0)} = (0,0)^t$. Then $\mathbf{x}^{(1)} =$

(a) $(0.4, 1.92)^t$
(b) $(1.6, 0.56)^t$
(c) $(0.4, 1.76)^t$
(d) $(-0.4, 1.76)^t$
(e) $(0.4, -1.44)^t$

18. Consider $f(x) = x - 10\pi + 6e$. Using three-digit rounding arithmetic, $f(-3/62) \approx$

(a) -15.2
(b) 15.0
(c) 14.8
(d) 14.9
(e) -15.1

19. Let $\mathbf{x} = (x_1, x_2)^t$ and $\mathbf{x}^{(1)}$ be the first iterate of the steepest descent method in solving the system of equations

$$3x_1^2 - x_2^2 = 0$$
$$3x_1x_2^2 - x_1^3 = 1,$$

with $\mathbf{x}^{(0)} = (1, \frac{1}{2})^t$, and $\alpha = 0.01$. Then $\mathbf{x}^{(1)} =$

(a) $(0.6138, 0.63)^t$
(b) $(0.6589, 0.57)^t$
(c) $(0.6138, 0.52)^t$
(d) $(0.7038, 0.51)^t$
(e) $(0.6589, 0.63)^t$

20. Consider the following data

x	0	1.0	2.0
y	0.2849	0.8934	β

Let $P_2(x) = a_2x^2 + 0.5255x + a_0$ be a least square polynomial fitted to the data. Then, $P_2(1.75) =$

(a) 2.0124
(b) 2.2457
(c) 1.8728
(d) 1.6679
(e) 1.4587

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE03 **CODE03**

Math 371
Final Exam
242
May 25, 2025
Net Time Allowed: 120 Minutes

Name	

ID		**Sec**	

Check that this exam has 20 questions.

Important Instructions:

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1. Consider the following data

x	1.0	2.0	3.0
y	2.3551	ℓ	3.1116

.

Let $f(x) = 2.1021e^{ax}$ by a least square curve fitted to the data. Then, $a =$

(a) 0.3782
(b) 1.1495
(c) -1.9711
(d) 0.1392
(e) -0.9723

2. The matrix $A = \begin{bmatrix} 2 & 1 \\ -1 & 1 \\ 1 & 1 \\ 2 & -1 \end{bmatrix}$ has the singular value decomposition $A = USV^t$, with

$$S = \begin{bmatrix} 3.1623 & 0 \\ 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad V = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

The first two columns of the matrix U obtained using S and V are:

(a) $\mathbf{u}_1 = (-0.6325, 0.3162, -0.3162, -0.6325)^t, \quad \mathbf{u}_2 = (0.5, 0.5, 0.5, 0.5)^t$
(b) $\mathbf{u}_1 = (0.6325, 0.3162, 0.3162, 0.6325)^t, \quad \mathbf{u}_2 = (0.5, 0.5, 0.5, 0.5)^t$
(c) $\mathbf{u}_1 = (0.6325, -0.3162, 0.3162, 0.6325)^t, \quad \mathbf{u}_2 = (0.5, 0.5, 0.5, -0.5)^t$
(d) $\mathbf{u}_1 = (0.6325, 0.3162, 0.3162, 0.6325)^t, \quad \mathbf{u}_2 = (-0.5, -0.5, -0.5, 0.5)^t$
(e) $\mathbf{u}_1 = (-0.6325, 0.3162, -0.3162, -0.6325)^t, \quad \mathbf{u}_2 = (-0.5, -0.5, -0.5, 0.5)^t$

3. Consider the linear system

$$\begin{aligned} 6x_1 + 3x_2 &= 4 \\ 2x_1 - 3x_2 &= 1 \end{aligned}$$

Let $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ be the first and second iteration, respectively, of the conjugate gradient method using the initial guess $\mathbf{x}^{(0)} = (1, -1)^t$ and the conjugate directions $\mathbf{v}^{(1)} = (1, 0)^t$ and $\mathbf{v}^{(2)} = (-1, 2)^t$. Then, $\|\mathbf{x}^{(1)} - \mathbf{x}^{(2)}\|_\infty =$

(a) 1.1667
(b) 1.000
(c) 1.0833
(d) 0.6250
(e) 0.5417

4. The linear finite difference scheme applied to the problem

$$y'' - y' - 2y = \cos(x), \qquad 0 \le x \le \frac{\pi}{2}, \qquad y(0) = -0.3,\ \ y(\frac{\pi}{2}) = -0.1,$$

with $\frac{\pi}{8}$ results in the system $Aw = b$, where

(a) $A = \begin{bmatrix} 2.3084 & -0.8037 & 0 \\ -1.1963 & 2.3084 & -0.8037 \\ 0 & -1.1963 & 2.3084 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.3836 \\ -0.1090 \\ -0.1786 \end{bmatrix}$

(b) $A = \begin{bmatrix} 2.3084 & -0.8037 & 0 \\ -1.1963 & 2.3084 & -0.8037 \\ 0 & -1.1963 & 2.3084 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.2228 \\ -0.1090 \\ -0.1786 \end{bmatrix}$

(c) $A = \begin{bmatrix} 2.3084 & -0.8037 & 0 \\ -1.1963 & 2.3084 & -0.8037 \\ 0 & -1.1963 & 2.3084 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.5014 \\ -0.1090 \\ -0.1394 \end{bmatrix}$

(d) $A = \begin{bmatrix} 1.6916 & -1.1963 & 0 \\ -0.8037 & 1.6916 & -1.1963 \\ 0 & -0.8037 & 1.6916 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.5014 \\ -0.1090 \\ -0.1394 \end{bmatrix}$

(e) $A = \begin{bmatrix} 1.6916 & -1.1963 & 0 \\ -0.8037 & 1.6916 & -1.1963 \\ 0 & -0.8037 & 1.6916 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.3836 \\ -0.1090 \\ -0.1786 \end{bmatrix}$

5. Let A be a real matrix with m rows and n column, $m \geq n$ and $A = USV^t$. Which of the following statements is not always true

   (a) U and V are orthogonal
   (b) S is invertible
   (c) All entries of S are non-negative
   (d) U is $m \times m$ and V is $n \times n$
   (e) U and V are always invertible

6. Consider the following data

x	0	1.0	2.0
y	0.2849	0.8934	β

   Let $P_2(x) = a_2x^2 + 0.5255x + a_0$ be a least square polynomial fitted to the data. Then, $P_2(1.75) =$

   (a) 1.6679
   (b) 1.4587
   (c) 2.2457
   (d) 2.0124
   (e) 1.8728

7. Using the composite trapezoidal rule with $n = 4$,

$$\int_0^2 \frac{1}{x+4}dx \approx$$

(a) 0.8124
(b) 0.2031
(c) 0.3914
(d) 0.4062
(e) 0.4055

8. Consider the problem

$$e^x - x^2 + 3x - 2 = 0, \quad -1 \leq x \leq 2.$$

The minimum number of iteration required by the bisection method to guarantee 10^{-8} accuracy is

(a) 35
(b) 29
(c) 25
(d) 27
(e) 15

9. Let $P_2(x)$ be the second Taylor polynomial for $f(x) = \ln(x+1)$ about $x_0 = 0$. Then the smallest upper bound, by the Taylor theorem, for $|f(0.5) - P_2(0.5)|$ is

   (a) 4.17×10^{-2}
   (b) 8.34×10^{-2}
   (c) 2.08×10^{-2}
   (d) 1.25×10^{-1}
   (e) 3.33×10^{-1}

10. Consider the boundary value problem

$$y'' = 4(y - x), \qquad 0 \le x \le 1, \qquad y(0) = 0,\ y(1) = 2.$$

   Applying the linear finite difference method with $h = \frac{1}{3}$

   (a) $y(\frac{1}{3}) = 0.3904,\ y(\frac{2}{3}) = 0.8060$
   (b) $y(\frac{1}{3}) = 1.0377,\ y(\frac{2}{3}) = 1.7623$
   (c) $y(\frac{1}{3}) = 0.2696,\ y(\frac{2}{3}) = 0.8073$
   (d) $y(\frac{1}{3}) = 1.7797,\ y(\frac{2}{3}) = 2.6203$
   (e) $y(\frac{1}{3}) = 0.5343,\ y(\frac{2}{3}) = 1.1580$

11. Using the secant method with $p_0 = -1$ and $p_1 = 0$ to approximate the solution of

$$-x^3 = \cos(x), \quad \text{we get} \quad p_3 =$$

(a) -0.8657
(b) -0.8803
(c) -1.2521
(d) -0.8655
(e) -0.7523

12. The singular values of the matrix $\begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$ are

(a) $s_1 = 7.1623, \quad s_2 = 0.8377$
(b) $s_1 = 2.6762, \quad s_2 = 0.8377$
(c) $s_1 = 1.2644, \quad s_2 = 0.7380$
(d) $s_1 = 2.6762, \quad s_2 = 0.9153$
(e) $s_1 = 2.5959, \quad s_2 = 1.1231$

13. Let $\mathbf{x} = (x_1, x_2)^t$ and $\mathbf{x}^{(1)}$ be the first iterate of the Newton's method in solving

$$4x_1^2 - 20x_1 + \frac{1}{4}x_2^2 = -8$$
$$-\frac{1}{2}x_1x_2^2 + 2x_1 - 5x_2 = -8,$$

with $\mathbf{x}^{(0)} = (0,0)^t$. Then $\mathbf{x}^{(1)} =$

(a) $(0.4, -1.44)^t$
(b) $(-0.4, 1.76)^t$
(c) $(0.4, 1.92)^t$
(d) $(1.6, 0.56)^t$
(e) $(0.4, 1.76)^t$

14. Consider $f(x) = x - 10\pi + 6e$. Using three-digit rounding arithmetic, $f(-3/62) \approx$

(a) -15.2
(b) -15.1
(c) 14.8
(d) 14.9
(e) 15.0

15. Consider the matrix A and the vector $\mathbf{b}$: $A = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 4 & -1 \\ -2 & -3 & 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}.$

    Let $A = LU$ be a factorization of A, where L is lower triangle matrix with 1 on its diagonal and U is upper triangle. The solution of system $L\mathbf{y} = \mathbf{b}$, where $\mathbf{y} = (y_1, y_2, y_3)^t$ is

    (a) $\mathbf{y} = (-3, 3, 1)^t$
    (b) $\mathbf{y} = (2, -5, 3)^t$
    (c) $\mathbf{y} = (-2, 5, -3)^t$
    (d) $\mathbf{y} = (0.1667, 0.667, 0.3333)^t$
    (e) $\mathbf{y} = (2, -5, -1)^t$

16. Let $P_2(x)$ be the interpolating polynomial for the data $(0, y), (1, 3), (2, 2)$. Suppose the coefficient of x^2 in $P_2(x)$ is 6, then $y =$

    (a) 10
    (b) 16
    (c) 4.5
    (d) 2.5
    (e) 8

17. Let $\mathbf{x} = (x_1, x_2)^t$ and $\mathbf{x}^{(1)}$ be the first iterate of the steepest descent method in solving the system of equations

$$
\begin{aligned}
3x_1^2 - x_2^2 &= 0 \\
3x_1x_2^2 - x_1^3 &= 1,
\end{aligned}
$$

with $\mathbf{x}^{(0)} = (1, \frac{1}{2})^t$, and $\alpha = 0.01$. Then $\mathbf{x}^{(1)} =$

(a) $(0.6589, 0.63)^t$
(b) $(0.6589, 0.57)^t$
(c) $(0.7038, 0.51)^t$
(d) $(0.6138, 0.52)^t$
(e) $(0.6138, 0.63)^t$

18. Using the Gram-Schmidt process to find a set of orthogonal vectors u_1, u_2, u_3 from the linearly independent vectors

$$\mathbf{v}_1 = (2, -1, 1), \quad \mathbf{v}_2 = (1, 0, 1), \quad \mathbf{v}_3 = (0, 2, 0),$$

gives

(a) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, \frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (\frac{2}{3}, \frac{2}{3}, -\frac{2}{3})^t$
(b) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, -\frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (2, -\frac{1}{3}, 1)^t$
(c) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, \frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (2, -\frac{1}{3}, 1)^t$
(d) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, \frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (-\frac{2}{3}, \frac{2}{3}, -\frac{2}{3})^t$
(e) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, -\frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (\frac{2}{3}, \frac{2}{3}, -\frac{2}{3})^t$

19. Consider the initial value problem

$$y' = y - t^2 + 1, \quad 0 \le t \le 2, \quad y(0) = 0.5.$$

Using the Euler method with $n = 4$, $y(1.0) \approx$

(a) 4.4375
(b) 1.6875
(c) 3.3750
(d) 2.2500
(e) 1.1250

20. The first iteration of the Gauss-Seidel method for solving the linear system

$$\begin{aligned} 2x_1 - x_2 &= 4 \\ -x_1 + 2x_2 - x_3 &= 9 \\ -x_2 + 2x_3 &= 6 \end{aligned}$$

using the initial guess $\mathbf{x}^{(0)} = (0, 0, 0)^t$ gives

(a) $\mathbf{x}^{(1)} = (2, 5.5, 3)^t$
(b) $\mathbf{x}^{(1)} = (2, 5.5, 5.75)^t$
(c) $\mathbf{x}^{(1)} = (2, 5.5, 3)^t$
(d) $\mathbf{x}^{(1)} = (4, 9, 6)^t$
(e) $\mathbf{x}^{(1)} = (2, 4.5, 3)^t$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE04 **CODE04**

Math 371
Final Exam
242
May 25, 2025
Net Time Allowed: 120 Minutes

Name	

ID		**Sec**	

Check that this exam has 20 questions.

Important Instructions:

1. All types of smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. Let $\mathbf{x} = (x_1, x_2)^t$ and $\mathbf{x}^{(1)}$ be the first iterate of the steepest descent method in solving the system of equations

$$3x_1^2 - x_2^2 = 0$$
$$3x_1x_2^2 - x_1^3 = 1,$$

with $\mathbf{x}^{(0)} = (1, \frac{1}{2})^t$, and $\alpha = 0.01$. Then $\mathbf{x}^{(1)} =$

(a) $(0.7038, 0.51)^t$
(b) $(0.6138, 0.52)^t$
(c) $(0.6138, 0.63)^t$
(d) $(0.6589, 0.63)^t$
(e) $(0.6589, 0.57)^t$

2. The first iteration of the Gauss-Seidel method for solving the linear system

$$2x_1 - x_2 = 4$$
$$-x_1 + 2x_2 - x_3 = 9$$
$$-x_2 + 2x_3 = 6$$

using the initial guess $\mathbf{x}^{(0)} = (0, 0, 0)^t$ gives

(a) $\mathbf{x}^{(1)} = (2, 4.5, 3)^t$
(b) $\mathbf{x}^{(1)} = (2, 5.5, 3)^t$
(c) $\mathbf{x}^{(1)} = (4, 9, 6)^t$
(d) $\mathbf{x}^{(1)} = (2, 5.5, 5.75)^t$
(e) $\mathbf{x}^{(1)} = (2, 5.5, 3)^t$

3. The linear finite difference scheme applied to the problem

$$y'' - y' - 2y = \cos(x), \qquad 0 \le x \le \frac{\pi}{2}, \qquad y(0) = -0.3,\ \ y(\frac{\pi}{2}) = -0.1,$$

with $\frac{\pi}{8}$ results in the system $Aw = b$, where

(a) $A = \begin{bmatrix} 2.3084 & -0.8037 & 0 \\ -1.1963 & 2.3084 & -0.8037 \\ 0 & -1.1963 & 2.3084 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.2228 \\ -0.1090 \\ -0.1786 \end{bmatrix}$

(b) $A = \begin{bmatrix} 2.3084 & -0.8037 & 0 \\ -1.1963 & 2.3084 & -0.8037 \\ 0 & -1.1963 & 2.3084 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.5014 \\ -0.1090 \\ -0.1394 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1.6916 & -1.1963 & 0 \\ -0.8037 & 1.6916 & -1.1963 \\ 0 & -0.8037 & 1.6916 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.3836 \\ -0.1090 \\ -0.1786 \end{bmatrix}$

(d) $A = \begin{bmatrix} 1.6916 & -1.1963 & 0 \\ -0.8037 & 1.6916 & -1.1963 \\ 0 & -0.8037 & 1.6916 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.5014 \\ -0.1090 \\ -0.1394 \end{bmatrix}$

(e) $A = \begin{bmatrix} 2.3084 & -0.8037 & 0 \\ -1.1963 & 2.3084 & -0.8037 \\ 0 & -1.1963 & 2.3084 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -0.3836 \\ -0.1090 \\ -0.1786 \end{bmatrix}$

4. The singular values of the matrix $\begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$ are

(a) $s_1 = 2.6762, \quad s_2 = 0.8377$

(b) $s_1 = 1.2644, \quad s_2 = 0.7380$

(c) $s_1 = 2.5959, \quad s_2 = 1.1231$

(d) $s_1 = 2.6762, \quad s_2 = 0.9153$

(e) $s_1 = 7.1623, \quad s_2 = 0.8377$

5. Using the composite trapezoidal rule with $n = 4$,

$$\int_0^2 \frac{1}{x+4}dx \approx$$

(a) 0.2031
(b) 0.3914
(c) 0.4062
(d) 0.4055
(e) 0.8124

6. Consider $f(x) = x - 10\pi + 6e$. Using three-digit rounding arithmetic, $f(-3/62) \approx$

(a) 14.8
(b) -15.2
(c) 14.9
(d) 15.0
(e) -15.1

7. Let $\mathbf{x} = (x_1, x_2)^t$ and $\mathbf{x}^{(1)}$ be the first iterate of the Newton's method in solving

$$4x_1^2 - 20x_1 + \frac{1}{4}x_2^2 = -8$$
$$-\frac{1}{2}x_1x_2^2 + 2x_1 - 5x_2 = -8,$$

with $\mathbf{x}^{(0)} = (0, 0)^t$. Then $\mathbf{x}^{(1)} =$

(a) $(1.6, 0.56)^t$
(b) $(0.4, 1.76)^t$
(c) $(0.4, -1.44)^t$
(d) $(-0.4, 1.76)^t$
(e) $(0.4, 1.92)^t$

8. Consider the following data

x	1.0	2.0	3.0
y	2.3551	ℓ	3.1116

Let $f(x) = 2.1021e^{ax}$ by a least square curve fitted to the data. Then, $a =$

(a) -1.9711
(b) 0.1392
(c) -0.9723
(d) 1.1495
(e) 0.3782

9. Consider the following data

x	0	1.0	2.0
y	0.2849	0.8934	β

Let $P_2(x) = a_2x^2 + 0.5255x + a_0$ be a least square polynomial fitted to the data. Then, $P_2(1.75) =$

(a) 2.0124
(b) 1.6679
(c) 2.2457
(d) 1.4587
(e) 1.8728

10. Consider the linear system

$$6x_1 + 3x_2 = 4$$
$$2x_1 - 3x_2 = 1$$

Let $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ be the first and second iteration, respectively, of the conjugate gradient method using the initial guess $\mathbf{x}^{(0)} = (1, -1)^t$ and the conjugate directions $\mathbf{v}^{(1)} = (1, 0)^t$ and $\mathbf{v}^{(2)} = (-1, 2)^t$. Then, $\|\mathbf{x}^{(1)} - \mathbf{x}^{(2)}\|_\infty =$

(a) 1.1667
(b) 0.5417
(c) 1.0833
(d) 1.000
(e) 0.6250

11. Let $P_2(x)$ be the second Taylor polynomial for $f(x) = \ln(x+1)$ about $x_0 = 0$. Then the smallest upper bound, by the Taylor theorem, for $|f(0.5) - P_2(0.5)|$ is

    (a) 1.25×10^{-1}
    (b) 4.17×10^{-2}
    (c) 8.34×10^{-2}
    (d) 2.08×10^{-2}
    (e) 3.33×10^{-1}

12. Using the secant method with $p_0 = -1$ and $p_1 = 0$ to approximate the solution of

    $-x^3 = \cos(x), \quad$ we get $\quad p_3 =$

    (a) -0.7523
    (b) -0.8655
    (c) -0.8803
    (d) -1.2521
    (e) -0.8657

13. Consider the initial value problem

$$y' = y - t^2 + 1, \quad 0 \le t \le 2, \quad y(0) = 0.5.$$

Using the Euler method with $n = 4$, $y(1.0) \approx$

(a) 1.6875
(b) 3.3750
(c) 4.4375
(d) 1.1250
(e) 2.2500

14. Using the Gram-Schmidt process to find a set of orthogonal vectors u_1, u_2, u_3 from the linearly independent vectors

$$\mathbf{v}_1 = (2, -1, 1), \quad \mathbf{v}_2 = (1, 0, 1), \quad \mathbf{v}_3 = (0, 2, 0),$$

gives

(a) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, \frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (-\frac{2}{3}, \frac{2}{3}, -\frac{2}{3})^t$
(b) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, -\frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (\frac{2}{3}, \frac{2}{3}, -\frac{2}{3})^t$
(c) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, -\frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (2, -\frac{1}{3}, 1)^t$
(d) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, \frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (2, -\frac{1}{3}, 1)^t$
(e) $\mathbf{u}_1 = (2, -1, 1)^t, \mathbf{u}_2 = (0, \frac{1}{2}, \frac{1}{2})^t, \mathbf{u}_3 = (\frac{2}{3}, \frac{2}{3}, -\frac{2}{3})^t$

15. Let A be a real matrix with m rows and n column, $m \geq n$ and $A = USV^t$. Which of the following statements is not always true

(a) U and V are always invertible
(b) U is $m \times m$ and V is $n \times n$
(c) U and V are orthogonal
(d) All entries of S are non-negative
(e) S is invertible

16. The matrix $A = \begin{bmatrix} 2 & 1 \\ -1 & 1 \\ 1 & 1 \\ 2 & -1 \end{bmatrix}$ has the singular value decomposition $A = USV^t$, with

$$S = \begin{bmatrix} 3.1623 & 0 \\ 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad V = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

The first two columns of the matrix U obtained using S and V are:

(a) $\mathbf{u}_1 = (-0.6325, 0.3162, -0.3162, -0.6325)^t, \quad \mathbf{u}_2 = (-0.5, -0.5, -0.5, 0.5)^t$
(b) $\mathbf{u}_1 = (0.6325, 0.3162, 0.3162, 0.6325)^t, \quad \mathbf{u}_2 = (-0.5, -0.5, -0.5, 0.5)^t$
(c) $\mathbf{u}_1 = (0.6325, 0.3162, 0.3162, 0.6325)^t, \quad \mathbf{u}_2 = (0.5, 0.5, 0.5, 0.5)^t$
(d) $\mathbf{u}_1 = (0.6325, -0.3162, 0.3162, 0.6325)^t, \quad \mathbf{u}_2 = (0.5, 0.5, 0.5, -0.5)^t$
(e) $\mathbf{u}_1 = (-0.6325, 0.3162, -0.3162, -0.6325)^t, \quad \mathbf{u}_2 = (0.5, 0.5, 0.5, 0.5)^t$

17. Consider the matrix A and the vector $\mathbf{b}$: $A = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 4 & -1 \\ -2 & -3 & 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}.$

    Let $A = LU$ be a factorization of A, where L is lower triangle matrix with 1 on its diagonal and U is upper triangle. The solution of system $L\mathbf{y} = \mathbf{b}$, where $\mathbf{y} = (y_1, y_2, y_3)^t$ is

    (a) $\mathbf{y} = (0.1667, 0.667, 0.3333)^t$
    (b) $\mathbf{y} = (2, -5, 3)^t$
    (c) $\mathbf{y} = (-3, 3, 1)^t$
    (d) $\mathbf{y} = (-2, 5, -3)^t$
    (e) $\mathbf{y} = (2, -5, -1)^t$

18. Let $P_2(x)$ be the interpolating polynomial for the data $(0, y), (1, 3), (2, 2)$. Suppose the coefficient of x^2 in $P_2(x)$ is 6, then $y =$

    (a) 8
    (b) 4.5
    (c) 2.5
    (d) 16
    (e) 10

19. Consider the boundary value problem

$$y'' = 4(y - x), \qquad 0 \le x \le 1, \qquad y(0) = 0,\ y(1) = 2.$$

Applying the linear finite difference method with $h = \frac{1}{3}$

(a) $y(\frac{1}{3}) = 0.5343,\ y(\frac{2}{3}) = 1.1580$
(b) $y(\frac{1}{3}) = 1.7797,\ y(\frac{2}{3}) = 2.6203$
(c) $y(\frac{1}{3}) = 0.2696,\ y(\frac{2}{3}) = 0.8073$
(d) $y(\frac{1}{3}) = 1.0377,\ y(\frac{2}{3}) = 1.7623$
(e) $y(\frac{1}{3}) = 0.3904,\ y(\frac{2}{3}) = 0.8060$

20. Consider the problem

$$e^x - x^2 + 3x - 2 = 0, \quad -1 \le x \le 2.$$

The minimum number of iteration required by the bisection method to guarantee 10^{-8} accuracy is

(a) 27
(b) 15
(c) 35
(d) 29
(e) 25

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	D 19	B 11	D 11	C 18
2	A	B 2	C 7	E 15	D 9
3	A	E 11	D 9	C 10	B 20
4	A	E 1	B 4	C 20	D 14
5	A	B 9	B 6	B 16	C 6
6	A	E 16	D 3	B 12	E 2
7	A	D 18	D 10	D 6	B 17
8	A	C 4	E 8	B 3	B 11
9	A	A 5	D 19	A 1	D 12
10	A	C 17	E 20	E 19	C 10
11	A	D 14	B 13	C 4	B 1
12	A	C 7	A 16	D 14	D 4
13	A	C 20	C 1	E 17	E 7
14	A	D 10	B 14	B 2	E 13
15	A	D 3	B 15	B 8	E 16
16	A	B 12	C 5	B 5	A 15
17	A	D 15	C 17	E 18	B 8
18	A	D 6	E 2	A 13	D 5
19	A	B 8	A 18	D 7	A 19
20	A	D 13	E 12	B 9	D 3

Answer Counts

V	A	B	C	D	E
1	1	4	4	8	3
2	2	6	4	4	4
3	2	7	3	4	4
4	2	5	3	6	4