Math405: Learning From Data
Final Exam

29th December 2021 at 7:00pm[a]

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[b] *Duration: 120 minutes*

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1. Cholesky Factorization (5 points)

Perform the Cholesky Factorization for $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$.

2. Gradient Descent (5 points)

Consider the problem where we have to:

$$\text{minimize } f(x_1, x_2) = (3x_1 - x_2)^2 + (2x_2 - 1)^2, \text{ starting from } (1, 1).$$

Perform manually one full iteration of the Newton-Raphson method.

3. Gram-Schmidt (5 points)

Find the Gram-Schmidt factorization of the matrix $A = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$.

4. Derivative of A^{-1} (5 points)

For$A(t) = \begin{pmatrix} 3t & 4t \\ 1 & 1 \end{pmatrix}$, find $\frac{\partial A^{-1}}{\partial t}$.

5. Generalized Eigenvectors (5 points)

For $S = \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}$, $M = \begin{pmatrix} 2 & 2 \\ 2 & 1 \end{pmatrix}$, and λ a generlized eigenvalue such that $S = \lambda M$, find a the form of the generalized eigenvectors X that solve $SX = \lambda MX$.

6. Least Squares (5 points)

For $A = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, find the least squares solution to $AX = B$.

7. Singular values (5 points)

For $A = \begin{pmatrix} 2t & 0 \\ -1 & 1 \end{pmatrix}$, find the singular values $\sigma_1(t)$ and $\sigma_2(t)$.

8. Matrix Completion (5 points)

a) Find the matrix $S = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$, with the smallest l_2 norm $||S||_2$ and such that:

$$S \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

b) Find the best possible real-valued matrix A^* that completes $A = \begin{pmatrix} * & 1 \\ 4 & * \end{pmatrix}$ and minimizes:

$$\min_{C \text{ and } R} \frac{1}{2}||(A - CR^t)_{known}||_2^2 + \frac{1}{2}(||C||_F^2 + ||R||_F^2)$$

where $CR^t = (2 \times 1)(1 \times 2)$.