Math405: Learning From Data
Exam 1

10th October 2022 at 6.30pm–8pm[a]

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Show Your Manual Work on Each Question

1. Cholesky Factorization (5 points)

Consider the matrix A:

$$A = \begin{pmatrix} 6 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

Perform the Cholesky factorization on A.

2. $\boldsymbol{A = X\Lambda X^{-1}}$ (5 points)

Consider the following matrix

$$A = \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}.$$

a. Find the eigenvalues λ of A.

b. Find orthonormal eigenvectors V of A.

c. Write the matrices X, X^{-1} and Λ such that: $S = X\Lambda X^{-1}$.

3. Positive Definite Matrices (5 points)

Consider the function $f(x, y) = x^2y - xy + y^2$.

a. Find every point (x, y) such that $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$.

b. Which of these point(s) is a minimum? Explain!

4. $S = Q\Lambda Q^t$ (5 points)

Consider the following matrix

$$S = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}.$$

a. Find the eigenvalues λ of S.

b. Find orthonormal eigenvectors V of S.

c. Write the matrices Q and Λ such that: $S = Q\Lambda Q^t$.

5. SVD (5 points)

Consider the matrix A such that:

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

Perform the Singular Value Decomposition such that $A = U_A \Sigma_A V_A^t$.