King Fahd University of Petroleum & Minerals
Department of Mathematics
Math407: Applied Game Theory
Midterm Exam
Four Questions, March 2nd, 2022 [1]

1   Short Questions (6 points)

For each of the following statements, **determine** whether it is true or false and **explain** your answer using a short paragraph, example or counter-example.

(a) A weakly dominated strategy can never be a best response. **(2 points)**

(b) Strategic form is the most complete way to model conflict situations. **(2 points)**

(c) A Nash equilibrium is a situation where every player gets always his or her absolute maximum payoff. **(2 points)**

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[1]This is an open book exam. The exam game lasts 80 minutes.

2   Party Game (8 points)

Player A has invited player B to his party. Player A must choose whether or not to hire a clown. Simultaneously, player B must decide whether or not to go to the party. Player B likes A but hates clowns (he even hates other people seeing clowns!) B's payoff from going to the party is 4 if there is no clown, but 0 if there is a clown there. B's payoff from not going to the party is 3 if there is no clown at the party, but 1 if there is a clown at the party. A likes clowns (he especially likes B's reaction to them) but does not like paying for them. A's payoff if B comes to the party is 4 if there is no clown, but $8 - x$ if there is a clown (x is the cost of a clown). A's payoff if B does not come to the party is 2 if there is no clown, but $3 - x$ if there is a clown there.

(a) Write down the payoff matrices of this game **(2 points)**.

(b) For which values of x player A would always avoid hiring a clown independently from player B's choice? Explain.**(3 points)**

(c) Suppose $x = 2$. Identify all dominated strategies. Explain. Is there any Nash equilibrium in pure strategies? **(3 points)**

3   Road Trip (8 points)

Six students are going on a foreign trip on which they will live close together. Where they are going, there is a disease which spreads easily among people who live close together. The value of the trip to a student who does not get the disease is 6. The value of the trip to a student who gets the disease is 0.

There is a vaccination against the disease. The vaccination costs different amounts for different students (perhaps they have different health plans). Let's call the students 1; 2; 3; 4; 5 and 6 respectively. The vaccination costs 1 for student 1; it costs 2 for student 2; etc.... If a student gets vaccinated, he will not get the disease. But, if he is not vaccinated then his probability of getting the disease depends on the total number in the group who are not vaccinated. If he is the only person not to get vaccinated then the probability that he gets the disease is 1/6. If there is one other person who is not vaccinated (i.e., two in all including him) then the probability that he gets the disease is 2/6. If there are two other people who are not vaccinated (i.e., three including him) then the probability that he gets the disease is 3/6, etc... For example, suppose only students 2 and 4 get vaccinated. Then 2's expected payoff is 6 -(2) where the 2 is the cost of the vaccination. Student 4's expected payoff in this case is 6 -(4). Student 5's expected payoff in this case (recall he did not get vaccinated) is $6 \times \frac{2}{6} + 0 \times \frac{4}{6} = 2$ where the fraction $\frac{4}{6}$ is the probability that he gets the disease. To model this game, suppose that each student aims to maximize his expected payoff.

The students decide, individually and simultaneously, whether or not to get a vaccination.

(a) Explain concisely whether or not it is a Nash equilibrium for students 1, 2, 3 and 4 to get vaccinated and students 5 and 6 not to get vaccinated. **(2 points)**

(b) Explain concisely whether or not it is a Nash equilibrium for students 1, 2 and 3 to get vaccinated and students 4, 5, and 6 not to get vaccinated. **(2 points)**

(c) Which players in this game have strictly or weakly dominated strategies? Explain your answers carefully including whether any domination is strict or weak. **(2 points)**

(d) If we delete all strictly and weakly dominated strategies from all players, which players in the game now have (iteratively) strictly or weakly dominated strategies? Explain carefully. **(2 points)**

4   Lend Money with No Regret (13 points)

Ali is deciding whether or not to lend money as a loan to his best friend Badr. Badr has a bad credit history. Badr has to decide whether or not to buy new furniture for his house. If he buys the furniture, he will be unable to repay the loan. If he does not buy, he will repay the loan. The payoffs in this game are as follows: if Ali refuses to lend money to Badr and Badr buys the furniture using a high interest bank loan, then Ali gets 0 and Badr gets -2. If Ali refuses to lend money to Badr and Badr does not buy, then Ali and Badr get 0. If Ali lends money to Badr and Badr buys, then Ali gets -2 and Badr gets 7. If Ali lends money to Badr and Badr does not buy, then Ali gets a payoff of 3 and Badr gets a payoff of 5.

(a) Suppose this game is played simultaneously. Use the Lemke & Howson algorithm to find a Nash equilibrium for this game.**(7 points)**

(b) Suppose this game is played sequentially with imperfect information and Badr plays first but after a probabiity p to not ask for a loan, where p is determined by external factors. **Draw** the game's tree and **label** carefully each node and information set. **Write** the sequential payoff matrices relative to this game. **(6 points)**