King Fahd University of Petroleum & Minerals
Department of Mathematics
Math407: Applied Game Theory
Final Exam
Three Questions, May 22^{nd}, 2022 [1]

1 Short Questions (10 points)

State whether each of the following statements is true or false **(2 point)**. For each, explain your answer in (at most) a short paragraph, example or counter-example **(3 points)**.

(a) A proper Nash equilibrium is always perfect.

(b) Every Correlated equilibrium is a Nash equilibrium.

[1] This is an open book exam. The exam game lasts 120 minutes.

2 Correlated equilibrium and Chicken Game (20 points)

Consider a version of the "Chicken game" described as follows. Like all forms of the chicken game, there are three Nash equilibria. The two pure strategy Nash equilibria are (D, C) and (C, D). There is also a mixed strategy equilibrium which results in expected payoffs of $14/3 = 4.667$ for each player.

	D	C
D	$0, 0$	$7, 2$
C	$2, 7$	$6, 6$

Now consider a third party (or some natural event) that draws one of three cards labeled: (C, C), (D, C), and (C, D). This exogenous draw event is assumed to be uniformly at random over the 3 outcomes. After drawing the card the third party informs the players of the strategy assigned to them on the card (but not the strategy assigned to their opponent).

(a) Suppose a player is assigned D. Supposing the other player played their assigned strategy, would he want to deviate? **(6 points)**

(b) Suppose a player is assigned C. Supposing the other player played their assigned strategy, would he want to deviate? **(6 points)**

(c) Find the third Nash equilibrium in mixed strategies. Does is represent a coordinated equilibrium of this game? **(8 points)**

3 Smugglers vs Customs (30 points)

An import-export business company (X) is operating in the borders region between two countries. The company ships containers from country A to country B. The containers are loaded on large trucks and travel long distances to pass the borders. Due to the high levels of economic borders' activity between the two countries, the customs officers (Y) of the two countries perform common coordinated random inspections of the merchandise loaded in each container. The company (X) has the opportunity to realize huge profits if it succeeds in shipping large quantities of a precious metal to country B. This illegal but very profitable activity is called **smuggling**.

If the customs (Y) do inspect and catch the company (X) smuggling, they impose a large penalty payment $P > 0$ for this crime. The company (X) has to choose between two strategies; Ship (S) or Not-Ship (NS). The customs have to choose between two strategies; Inspect (I) or Not to inspect (NI). If the customs play (I) and the company plays (S), the customs get a payoff of P and the company gets a payoff of $-P$. If the customs play (I) and the company plays (NS), the customs get a payoff of -10 and the the company gets a payoff of 0. If the customs play (NI) and the company plays (S), the customs get a payoff of -100 and the company gets a payoff of 500. Finally, if the customs plays (NI) and the company play (NS), the customs get a payoff of 0 and the consumer gets a payoff of 0.

(a) For which values of P none of the pure strategies of (X) and (Y) is strictly dominated by the other. **(5 points)**

(b) Assume that $P = 1000$. Use the Lemke & Howson algorithm to find a Nash equilibrium for this game. Is your Nash equilibrium quasi-strong? Isolated? Regular? **(3+3+2+2 = 10 points)**

(c) Discuss the impact of the change in the value of P on the strategies played by (X) and (Y). **(5 points)**