

King Fahd University of Petroleum & Minerals
Department of Mathematics
Math407: Applied Game Theory
Midterm Exam
Four Questions, March 18th, 2023 ¹

1 Short Questions (5 points)

For each of the following statements, **determine** whether it is **TRUE** or **FALSE**.

- (a) A weakly dominated strategy can never be a best response. **(1 point)**
- (b) Strategic form is the most complete way to model conflict situations. **(1 point)**
- (c) A Nash equilibrium is a situation where every player gets always his or her absolute maximum payoff whatever the other players do. **(1 point)**
- (d) A Nash equilibrium could be Pareto efficient or Pareto inefficient. **(1 point)**
- (e) Regardless of the the conflict to be analyzed, a Game is only designed to get a Nash equilibrium. **(1 point)**

¹This is an open book exam. The exam game lasts 120 minutes.

2 Khaled lending money (5 points)

Khaled and Badr are very close friends. Khaled is deciding whether or not to lend money to Badr. Badr has a bad credit history with Khaled, but he needs money to end his month. Secretly, Badr has also to decide whether or not to buy a new mobile phone. If he buys the phone, he will be unable to repay the loan next month. If he does not buy, he will repay the loan. The payoffs in this game are as follows: if Khaled refuses to lend money to Badr and Badr buys the phone using a high interest credit card, then Khaled gets -1 and Badr gets -5 . If Khaled refuses to lend money to Badr and Badr does not buy, then Khaled gets 0 and Badr gets -1 . If Khaled lends money to Badr and Badr buys, then Khaled gets -3 and Badr gets 5 . If Khaled lends money to Badr and Badr does not buy, then Khaled gets a payoff of 1 and Badr gets a payoff of 3 .

(a) Write down the bimatrix payoffs for this game. Is there any strictly or weakly dominated strategy? **(0.5+0.5=1 point)**

(b) Suppose this game is played sequentially with imperfect information and Badr plays first. Badr has a probability p to buy, where p is determined by his responsiveness to external factors like peer pressure and intensive exposure to social media advertising campaigns.

i) **Draw** the game tree and **label** carefully each **node** and **information set**. **(1+1+1=3 points)**

ii) **Write** the sequential payoff matrices relative to this game. **(1 point)**

3 Oil Distribution Problem (9 points)

Two countries (A) and (B) are about to settle a long term oil-supply agreement. Two possibilities to ship oil from (A) to (B) are on the table. The first possibility uses oil tanker ships. The second possibility uses oil pipelines.

Oil tankers shipped from (A) can dock in many seaports of country (B). Oil is then transported to existing old refineries. Oil tankers and old refineries have an important environmental impact. The increasing awareness in country (B) is lobbying against this possibility.

There are currently no oil pipelines from country (A) to country (B). If this option is chosen, pipelines would need to be installed and countries (A) and (B) would need to share the costs. Besides, countries (A) and (B) would need to negotiate a deal with other countries crossed by the oil pipelines. Country (B) would also need to build new refineries with lower environmental impact in the regions where oil would be delivered.

If (A) and (B) agree on a long term oil supply contract using oil tankers ships only, the payoff for country (A) is estimated to 15 Billion dollars and the payoff for country (B) is estimated to 5 Billion dollars. If (A) insists on using oil tankers only, while country (B) rejects oil tankers shipments and decides to invest in oil pipelines and new refineries alone, the payoff for country (A) is estimated to 0 Billion dollars and the payoff for country (B) is estimated to -10 Billion dollars.

If (A) and (B) agree on a long term oil supply contract using oil pipelines only, the payoff for country (A) is estimated to 20 Billion dollars and the payoff for country (B) is estimated to 10 Billion dollars. If (A) insists on investing on oil pipelines alone, while country (B) decides to receive oil tankers shipments only and to keep using its old refineries, the payoff for country (A) is estimated to -10 Billion dollars and the payoff for country (B) is estimated to -10 Billion dollars.

(a) Write down the bimatrix payoffs for this game. Is there any strictly or weakly dominated strategy? **(0.5+0.5=1 point)**

(b) Write down the primal and dual linear programs for each of the two countries (A) and (B). **(0.5+0.5=1 point)**

(c) Use the Lemke & Howson algorithm to find a Nash equilibrium for this game. Is your Nash equilibrium in pure strategies? **(4 points)**

(d) Find a different Nash equilibrium (in pure strategies) for this game. **(1 point)**

(e) Is any of the Nash equilibria you obtained in (c) and (d) Pareto efficient? Which Nash equilibrium do you think is the “best”? Explain. **(0.5+0.5=1 point)**

(f) Is the strategy profile $(\frac{4}{7}, \frac{3}{7}) \times (\frac{4}{9}, \frac{5}{9})$ a Nash equilibrium for this game? **(1 bonus point)**

4 Two Retailers vs. Manufacturer (8 points)

A conflict involves a manufacturer and two retailers of a given electronic device. The retailers buy the device at the same price w from the manufacturer. They sell it on the market at their chosen prices p_1 and p_2 . The market share of each retailer depends on the price p_1 or p_2 , and the common warranty period t offered by the manufacturer. We assume that the market share is $D_1 = 2000 - 3p_1 + t$ for retailer 1, and $D_2 = 1500 - 2p_2 + t$ for retailer 2.

The manufacturer has no control on his unit production cost $c = 150$ (in dollars) of the electronic device. However, he controls the price w he bills to the retailers and the warranty period t (in months). We assume that the manufacturer's total warranty cost is $\omega(t) = -t^2 - tw$.

(a) If the two retailers sign an agreement to sell the electronic device at exactly the same price p that maximizes their total profit ($R(p) = R_1(p) + R_2(p)$), find the Nash equilibrium values p^* , w^* and t^* of the game involving the manufacturer vs. the two retailers playing as a single one. **(3 points)**

(b) If the two retailers do not sign any agreement and decide to sell the electronic device at p_1 and p_2 that maximize their respective profits ($R_1(p_1)$ and $R_2(p_2)$), find the Nash equilibrium values p_1^{**} , p_2^{**} , w^{**} and t^* of the game involving the manufacturer vs. the two retailers. **(4 points)**

(c) Comparing your results in (a) and (b), discuss which retailer has more interest to sign the agreement in (a), and whether or not he may offer to the other retailer any reasonable percentage of the total profit to make him agree? **(1 bonus point)**