King Fahd University of Petroleum & Minerals
Department of Mathematics
Math 407 - Applied Game Theory
Final Exam
Three Questions, May 27^{th}, 2023 [1]

1 Fast-Food Delivery Competition Game (10 points)

Figure 1 illustrates an extensive competition game with perfect information involving three fast-food delivery Apps. Each of the delivery Apps 1, 2 and 3, has to decide either to enter the market zones of both of its opponents or not. Hence, each delivery App randomizes on two pure strategic decisions "In" and "out". Each delievery App gets a partial payoff depending on its decision and the opponents' decisions. For example, if delivery App 1 decides to get "In" while delivery Apps 2 and 3 decide to get in, App 1 gets $1 + 2 = 3$ as a total payoff and Apps 2 and 3 get, respectively, $-1 + 3 = 2$ and $-1 + 6 = 5$.

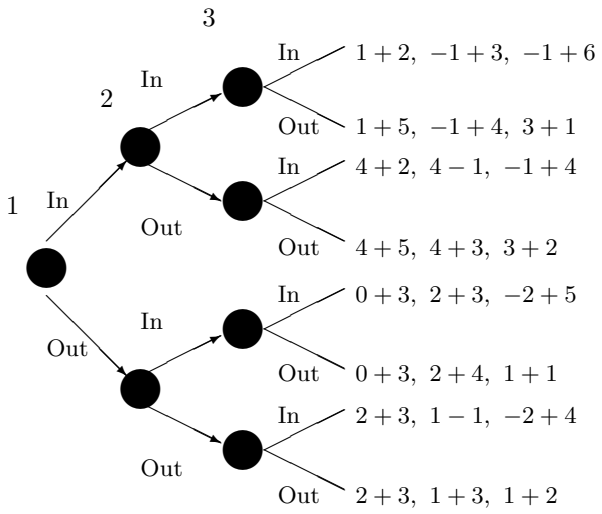

Figure 1: Delivery-App Competition Game

Question: Assuming perfect information, find a sequentially rational Nash equilibrium for the delivery-App competition game.

[1] The exam lasts 120 minutes.

2 Equilibrium Refinement (10 points)

For the following bimatrix game, we have enumerated 5 extreme Nash equilibria in Table 1.

$$A = \begin{pmatrix} -1 & -3 & 1 & 4 \\ 3 & 3 & 3 & 1 \\ 2 & -3 & 3 & -3 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 1 & 2 & 3 \\ -1 & -2 & 3 & 2 \\ 2 & 3 & -1 & 3 \end{pmatrix}.$$

Eq.	x_1			x_2				α_1	α_2
1	0	1	0	0	0	1	0	3	3
2	0	4/5	1/5	0	0	1	0	3	11/5
3	1	0	0	0	0	0	1	4	3
4	1	0	0	3/7	0	0	4/7	13/7	3
5	1/2	1/2	0	0	0	3/5	2/5	11/5	5/2

Table 1: Extreme Nash Equilibria

(a) Enumerate all maximal Nash subsets. **(3 points)**

(b) Which Extreme Nash equilibria are Quasi-strong? **(3 points)**

(c) Which Extreme Nash equilibria are regular? **(1 points)**

(d) Which Extreme Nash equilibria are NOT perfect? **(3 points)**
(Hint: Detect which strategies are weakly dominated for each player.)

3 Duopoly Game (10 points)

Two Oil producing countries determine their best reponses production q_1 and q_2. The market's price $p = f(q_1, q_2)$ is a function of the quantities produced. Each country aims at maximizing its own profit $\Pi_i(q_i, q_j) = pq_i - C_i(q_i)$, $i \neq j$. One can assume that the price writes as $p = a - b(q_1 + q_2)$ and the production cost writes as $C_i = c_i q_i$, where $a = 40$, $b = 1$ and $c_1 = c_2 = 10$ are contant paramters. The goal of this game is to determine the equilibrium market price.

(a) For the Cournot duopoly model, determine the best response quantities produced, the equilibrium market price p^* and the profits of the two countries.

(b) For the Stackelberg duopoly model, assume that country 1 is the leader and country 2 is the follower. Determine the best response quantities produced, the equilibrium market price p^{**} and the profits of the two countries.

(c) Compare the two models equilibrium market prices p^* and p^{**}, the total quantities produced and the profits of the two countries. Conclude on which model should be applied for the best of each of the two countries.