KFUPM/ Department of Mathematics/211/MATH 423/Exam 1

Name: ID#:

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1. [10pts] (a) Show that every f.g. abelian group is a homomorphic image of a f.g. free abelian group.

(b) Show that the abelian group $(\mathbb{R}, +)$ is not f.g.

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2. [10pts] For every abelian group G, let $T(G)$ denote its torsion subgroup (i.e. the subgroup consisting of all elements of finite order).

(a) Prove that $T(G/T(G)) = 0$.

(b) Is it true that each homomorphic image of a torsion-free abelian group is torsion-free? Justify.

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3. [10pts] (a) Is S_3 is solvable? Justify.

(b) Prove that for any group G, if $G/Z(G)$ is solvable, then G is solvable.

(c) Let G be a group with derived subgroup G' and let $K \lhd G$ be such that $K \cap G' = \{1\}$. Prove that $K \leq Z(G)$.

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4. [10pts] (a) Is the quaternion group Q_8 nilpotent? Justify.

(b) Give an example of an infinite non-abelian nilpotent group.