King Fahd University of Petroleum and Mineral College of Computing and Mathematics Department of Mathematics

# MATH436 – Discrete Models

Academic Year 2024-25

Term 242

# Midterm Exam

Name:	
ID#:	

Question	Score	Max Score
1		10
2		10
3		15
4		10
5		10
6		10
7		10
Total		75

Time allowed: 100 Minutes

## **Question 1**

The differential equation  $\frac{d^2y}{dx^2} + 2y = 0$  has a power series solution  $y = \sum_{k=0}^{\infty} c_k x^k$ .

Determine the difference equation satisfied by  $c_k$ .

### **Question 2**

- (a) Plot the curve  $y_{k+1} 1 = (y_k 1)^2$
- (**b**) Find the fixed points
- (c) Find the exact solution by setting  $x_k = \ln(y_k 1)$

Question 3 Solve the difference equations

(a)  $y_{k+1} + 4^k y_k = 0$ (b)  $y_{k+1} - y_k = 1 + e^k$ 

Question 4 Consider the difference equation  $y_{k+2} - (k+3)(k+2)y_k = 0$ Given that  $y_k^{(1)} = (k+1)!$  is a solution, find a second linearly independent solution.

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#### **Question 5**

(a) Find the general solution of the homogeneous equation

$$y_{k+3} + 5y_{k+2} + 3y_{k+1} - 9y_k = 0$$

(b) Write the form of the particular solution of the inhomogeneous equation

$$y_{k+3} + 5y_{k+2} + 3y_{k+1} - 9y_k = k(-3)^k + 2$$

Question 6 Solve the initial value problem

$$\Delta^3 y_k = 0, \qquad y_0 = 0, \qquad y_1 = 2, \qquad y_2 = 1$$

**Question 7** Solve the difference equation

$$y_{k+2} - 7y_{k+1} + 10y_k = 2k + 3^k$$