

King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 445 Major Exam I
The Second Semester of 2021-2022 (212)

Time Allowed: 90 Minutes

Name: _____ ID#: _____

Section/Instructor: _____ Serial #: _____

- Mobiles and calculators are not allowed in this exam.
 - Provide all necessary steps required in the solution.
 - Attempt all questions to the point.
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Question #	Marks	Maximum Marks
1		$5 + 5 = 10$
2		$4 + 3 = 7$
3		$9 + 6 = 15$
4		$4 + 4 + 4 = 12$
5		$4 + 4 + 4 + 4 = 16$
Total		60

Full Exam paper

1. (a) Draw the hexagon inscribed in a unit circle whose vertices represent the complex numbers z such that $z^6 = 1$.
(b) Evaluate $(-\sqrt{3} - i)^{29}$.
2. (a) Define domain and region in the complex plane.
(b) Is the set $3 \leq |z| < 11$ a domain or region or both? Give reason.
3. (a) Let $f(z) = z + 1$, $g(z) = e^{i\frac{\pi}{4}}z$ and $h(z) = 2z$ then sketch the image of the unit semi-disk $|z| \leq 1$, $\text{Im}(z) > 0$ under the following maps.
 - i. $h(z)$
 - ii. $g(h(z))$
 - iii. $f(g(h(z)))$(b) By using $\epsilon - \delta$ definition of limit show that $\lim_{z \rightarrow i} (2z + 1) = 1 + 2i$ by finding δ in terms of ϵ .
4. Determine the points where the following functions are differentiable and analytic.
 - (a) $f(z) = \bar{z}$
 - (b) $f(z) = (3x^2 + y + 3) + i(y^2 - x)$
 - (c) $f(z) = (3x^2 + 2x - 3y^2 - 1) + i(6xy + 2y)$
5. Write down T for a true and F for a false statement by supporting your answer with appropriate reason.
 - (a) Let f be an analytic function on $D := \{|z| < 1\} \cup \{|z| > 1\}$ such that $f'(z) = 0$. Then f is constant on D .
 - (b) The function $u(x, y) = e^{-x} \sin y$ can be real part of some analytic function.
 - (c) $f(z) = \frac{|z| + z}{2}$ is analytic at the origin.
 - (d) Let $f(z) = 3 + iv(x, y)$ be an analytic function then $v(x, y)$ must be constant.
 - (e) The curves $3x^2 + 2x - 3y^2 - 1 = c_1$ and $6xy + 2y = c_2$ are level curves for arbitrary constant c_1 and c_2 .