King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 445 Major Exam 2
The Second Semester of 2021-2022 (212)
Time Allowed: 90 Minutes

Name: _____ ID#: _____

Section/Instructor: _____ Serial #: _____

- Mobiles and calculators are not allowed in this exam.
- Provide all necessary steps required in the solution.
- Attempt all questions to the point.
- No credit for answers without justification.

Question #	Marks	Maximum Marks
1		
2		
3		
4		
5		
6		
Total		60

Full Exam paper

1. (a) Find the principal value of $\left(\frac{e}{2}\left(-1+i\sqrt{3}\right)\right)^{5\pi i}$.

   (b) Determine and sketch the domain of analyticity of $f(z) = \text{Log}(39+i-z)$. Calculate $f'(z)$.

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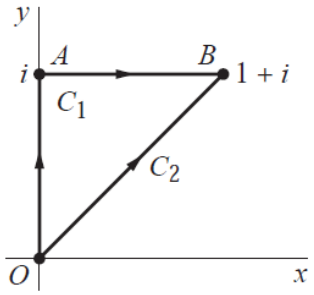
2. (a) Identify the interior of the simple closed contour given in the figure. Is it positively oriented?

   (b) Evaluate $\int_C f(z)dz$ where $f(z) = y - x - i3x^2$ and C is the contour given in the following figure.



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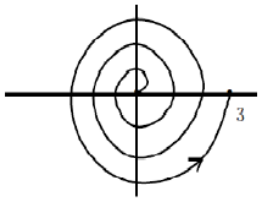
3. (a) Show that $\left|\int_C \frac{1}{3+5z^3}dz\right| \leq \pi$ where $C : e^{it}, 0 \leq t \leq 2\pi$.

   (b) Evaluate $\int_C (z^2-2)dz$ where C is the contour given in the following figure.



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4. (a) State the definition of simply and multiply connected domains. Given an example of each type.

   (b) If Γ is upper semi-circle from $z = 1$ to $z = -1$ and z^i is the principal value then show that

$$\int_\Gamma \frac{z^i}{1-i}dz = \frac{1}{2}(1+e^{-\pi})$$

(c) State the Cauchy inequality theorem for Cauchy estimates.

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5. (a) Using the Cauchy integral formula, compute

$$\oint_{|z|=\sqrt{5}} \frac{dz}{(z-i)^2(z-3i)^2}$$

(b) Let $f(z) = \sum_{j=0}^{\infty} \frac{j^3}{3^j} z^j$ then compute

$$\int_{|z|=1} \frac{f(z)\cos z}{z^2} dz$$

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6. Write down T for a true and F for a false statement by supporting your answer with an appropriate reason.

(a) 1 raise to any power is not always equal to 1.

(b) Let f be an analytic function, inside and on $|z| = 3$. Then f can not attain its max value at $z = \frac{5}{4} + i\frac{\sqrt{119}}{4}$.

(c) Let $f(z) = \sum_{j=0}^{\infty} \frac{f^{(j)}(39\pi)}{j!}(z-39\pi)^j$ and $g(z) = \sum_{j=0}^{\infty} \frac{g^{(j)}(1-i\pi)}{j!}(z-1+i\pi)^j$ be Taylor series expansions of $f(z)$ and $g(z)$. Then it is possible that $f(z) = g(z)$. If the statement is true, give an example of such a function.