

King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 445 Final Exam
The Second Semester of 2021-2022 (212)

Time Allowed: 150 Minutes

Name: _____ ID#: _____

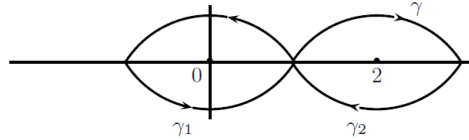
Section/Instructor: _____ Serial #: _____

- Mobiles and calculators are not allowed in this exam.
 - Provide all necessary steps required in the solution.
 - Attempt all questions to the point.
 - No credit for answers without justification.
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Question #	Marks	Maximum Points
1	8+7	
2	5+10	
3	3+9	
4	3+10	
5	3+10	
6	4+4+4	
Total		80

Full Exam paper

1. (a) Determine the domain of differentiability D of $f(z) = \bar{z}e^{-|z|^2}$ and compute $f'(z)$ at D , if it exists. Find all points where f is analytic.
- (b) Compute $\int_{\gamma} \frac{e^z}{z(z-2)} dz$, where γ is the following contour.



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2. (a) Given an example of a function which has removable singularity at $z = 0$, a pole of order 5 at $z = 1$, and an essential singularity at $z = i$.
- (b) Find the Laurent series representation for the function $\frac{z}{(z+1)(z-2)}$ in each of the following domains.
- $|z| < 1$
 - $1 < |z| < 2$
 - $|z| > 2$

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3. (a) State the Cauchy's residue theorem.
- (b) Evaluate by using residue theorem

$$\oint_{|z|=1} (3 - 2z + 4z^2)e^{-1/z} dz$$

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4. (a) State the Jordan's Lemma.
- (b) Compute

$$\text{P.V.} \int_{-\infty}^{\infty} \frac{\cos(3x)}{x^2 + 11} dx$$

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5. (a) State the Rouché's theorem.
- (b) By using the Rouché's theorem, show that all zeros of the $f(z) = z^6 - 5z^2 + 10$ lies in the annulus $1 \leq |z| \leq 2$. Also find the winding number of $f(z)$.

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6. Write down T for a true and F for a false statement by supporting your answer with an appropriate reason.
- Let $f(z) = z - \sin z$ then the angle between any two curves intersecting at $f(0)$ is twice the angle of intersection of these curves at $z = 0$.
 - Let $f(z) = e^{1/z}$ and $D := \{z : 0 < |z - 0| < \frac{1}{3}\}$ be a neighbourhood of 0. Then image of D under f is equal to the whole complex plane except 0.
 - Let D and D' be two domains then there always exist a one-to-one mapping from D to D' .