

King Fahd University of Petroleum and Minerals
Department of Mathematics
MATH445 - Intro. to Complex Variables
Major Exam II – Semester 221

Exercise 1

1. Find all numbers $z \in \mathbb{C}$ such that

(a) $e^{iz} = 1 + i$

(b) $\sin z = \cos z$

Exercise 2

Let $S = \{x + iy \mid 0 < x < \pi \text{ and } y \in \mathbb{R}\}$ and $g(z) = e^{iz}$. Show that g is one-to-one on S and find $g(S)$.

Exercise 3

1. Determine the inverse of the function

$$w = q(z) = e^{2z} - 2e^z$$

explicitly in terms of the complex logarithm.

2. Find all values of z for which $q(z) = i$.

Exercise 4

1. Compute $\int_C \bar{z} dz$ where C is the circle $|z| = 1$ traced once counterclockwise.
2. Compute $\int_\Gamma \operatorname{Im} z dz$ along the directed line segment from $z = 0$ to $z = 1 + 2i$.

Exercise 5

1. Evaluate

$$I_r := \int_{\Gamma_r} \frac{dz}{(z-2)(z+1)}$$

where Γ_r is the circle $|z| = r$ traced twice in the clockwise direction, with $r > 0$ and $r \neq 1, r \neq 2$.

2. Let P be a polynomial of degree at least 2 and P has all its zeros inside the circle $|z| = r$. Show that

$$\oint_{|z|=r} \frac{dz}{P(z)} = 0.$$

Exercise 6

Suppose that f is analytic on the closed disk $|z| \leq 1$. Let

$$G(z) = \frac{1}{2\pi i} \oint_{|w|=1} \frac{f(w)/w^2}{w-z} dw$$

- (a) Show that G is analytic on the unit disc.
- (b) Use partial fractions to express G as a function of f .

Exercise 7

Suppose that f is entire such that

$$|f(z)| \leq |z^3 - 1| \text{ for all } z \in \mathbb{C}.$$

Show that $f(z) = c(z^3 - 1)$, where $c \in \mathbb{C}$, with $|c| \leq 1$.