

MATH 445 (TERM 231) MIDTERM

CAUTION: Write down your answers as clear as possible.

1. Find the **principal value** of

$$\left[\frac{e}{2}(-1 - \sqrt{3}i) \right]^{3\pi i}.$$

2. Describe the image of $\{z \in \mathbb{C} : |z| = 1\}$ by the map

$$w = \frac{4(z+1)}{3+z},$$

3. Prove that a analytic function f on a domain D is a constant of $(\operatorname{Re} f)^2 - (\operatorname{Im} f)^2 \equiv 1$.

4. Prove that

$$u(x, y) = \ln(4x^2 + 4y^2 + 4x + 1) + x^2 - y^2$$

is harmonic and find its harmonic conjugate

5. Find all values of

$$(-\sqrt{3} + i)^{-1/3}.$$

6. Let $H = \{z \in \mathbb{C} : \operatorname{Im} z > 0\}$ be the upper half plane.

(a) Prove that if $z \notin H$ and $w \in H$, then $\frac{1}{z-w} \in H$.

(b) Let $P(z)$ be a polynomial of degree $k \geq 2$. Prove that if all the zeros of P lie on H , then so are zeros of P' . (Hint. Let $z_1, \dots, z_k \in H$ be zeros of P . Using the formula for $P'(z)/P(z)$, derive a contradiction if there is a zero z_0 of P' which does not lie on H from $P'(z_0)/P(z_0) = 0$.)