MATH 451 MIDTERM EXAM (TERM 211)

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1. Reparmetrize $\alpha(t) = (\cosh t,\, \sinh t,\, t)$ as a unit speed curve.

2. Prove that

$$\alpha(s) = \left(\frac{\cos^{-1} s - s\sqrt{1-s^2}}{2}, \frac{1-s^2}{2} \right), \quad -1 < s < 1$$

is a unit speed curve. Find $T(s)$, $N(s)$ and the curvature $\kappa(s)$. (Hint. Use $\frac{d}{ds}(\cos^{-1} s) = -\frac{1}{\sqrt{1-s^2}}$.)

3. Let α be a unit speed curve. Prove

$$(\alpha' \times \alpha'') \cdot \alpha''' = \kappa^2 \tau.$$

4. Show

$$\alpha(t) = (6t, 3t^2, t^3)$$

is a cylindrical helix, that is, there is a constant unit vector $\vec{u}$ whose angle with T is constant. What is the angle between $\vec{u}$ and T?

5. Let $\alpha(s)$ be a unit speed curve with constant curvature $\kappa \neq 0$. Suppose α is a cylindrical helix. Let $\vec{u}$ be the unit vector which has a constant angle θ with the tangent vector T of α. Assume by a change of coordinates that $\vec{u} = e_3 = (0,0,1)$ and let $\alpha(s) = (x(s), y(s), z(s))$. Then show that $\beta(s) = (x(s), y(s))$ is a circle. (s is not necessarily the arclength paramenter of β.) Express the radius of the circle in terms of κ and θ.

6. Let

$$\mathbf{x}(u,v) = (\cos u - v \sin u,\, \sin u + v \cos u,\, v)$$

be the parametrization of the hyperboloid $x^2 + y^2 - z^2 = 1$ as a ruled surface. Find the matrix representation of the shape operator S_p with respect to the basis $\{\mathbf{x}_u, \mathbf{x}_v\}$ at $p = \mathbf{x}(\frac{\pi}{2}, 0) = (0,1,0)$.