MATH 451 FINAL EXAM (TERM 211)

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1. Compute the Gauss and the mean curvatures of the graph of the function

$$z = \frac{x^4 + y^4}{4}$$

at $(x, y) = (1, 1)$.

2. Let $\alpha(v) = (f(v), g(v))$ be a unit speed curve. Suppose α is 1-1 and $f(v) > 0$. Let

$$\mathbf{x}(u, v) = (f(v)\cos u,\ f(v)\sin u,\ g(v) + cu), \quad c \text{ is a constant.}$$

Prove that $\mathbf{x}$ is either a surface of revolution or the helicoid if $F = \mathbf{x}_u \cdot \mathbf{x}_v = 0$.

3. Prove that the at a given point p on a surface M, the sum of normal curvatures in any two orthonormal directions is constant.

4. Prove that a Monge patch $\mathbf{x}(u, v) = (u, v, f(u, v))$ is a minimal surface if and only if

$$(1 + f_u^2)f_{vv} - 2f_u f_v f_{uv} + (1 + f_v^2)f_{uu} = 0.$$

5. Let $\gamma(s)$ is a unit speed geodesic on a surface M. Assume that the curvature κ of γ never vanishes. Prove that γ is a plannar curve if it is a line of curvature in M.

6. Let

$$\mathbf{x}(u, v) = (f(v)\cos u,\ f(v)\sin u,\ g(v))$$

be a surface of revolution of a unit speed curve $\alpha(v) = (f(v), g(v))$ $(f(v) > 0)$.

(a) Show that every meridian (that is, the v-parametric curve) is a gedesic.

(b) Under which condition on α at $v = v_0$, the u-parametric curve $\beta(u) = \mathbf{x}(u, v_0)$ is a geodesic?