King Fahd University of Petroleum & Minerals
Department of Mathematics
MATH 467: Graph Theory

Exam Duration: 90 minutes
Exam Weight: 25%

Student Name: _____

Student ID: _____

Instructions

- This exam consists of 10 multiple-choice questions and 1 problem-solving question.

- Each multiple-choice question is worth 2.5%, and the BONUS problem-solving question is worth 5%.

- Choose the correct answer for each multiple-choice question.

- No calculators, phones, or electronic devices are allowed.

- You have 90 minutes to complete this exam.

Multiple-Choice Questions (Each 2.5%)

1. A graph G of order 26 and size 58 has 5 vertices of degree 4, 6 vertices of degree 5, and 7 vertices of degree 6. The remaining vertices of G all have the same degree. What is this degree?

    (A) 3   (B) 4   (C) 5   (D) 6   (E) 7

2. How many graphs are there with 5 vertices that have every vertex degree distinct?

   (A) 0   (B) 1   (C) 2   (D) 3   (E) 4

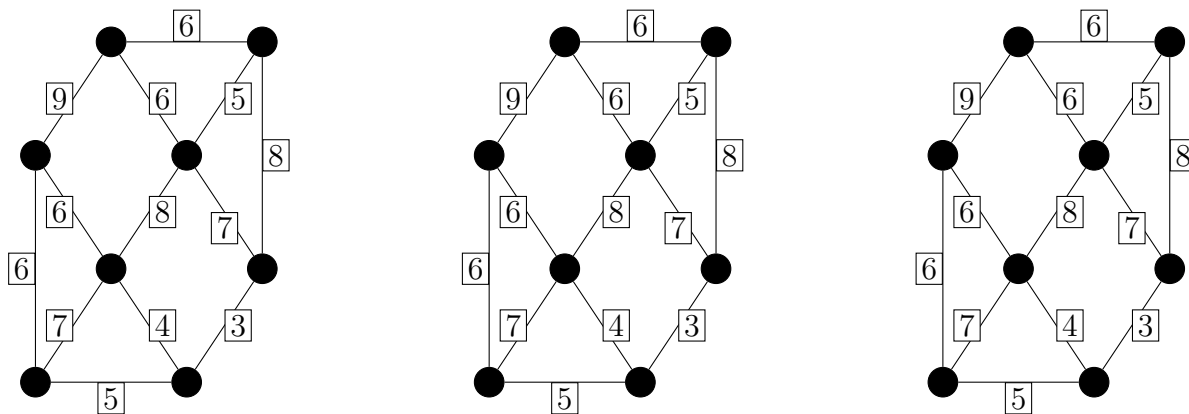3. Determine the order and size of $P_3 \square 2P_3$.

    (A) Order: 9, Size: 18   (B) Order: 12, Size: 24   (C) Order: 18, Size: 36
    (D) Order: 24, Size: 48   (E) Order: 36, Size: 72

4. (A) Consider the graph shown above ("The graph is repeated three times for your convenience, providing extra room for you to work on your solutions."). What is the maximum weight spanning tree of this graph?

(A) 49   (B) 50   (C) 52   (D) 54   (E) 57

(B) How many maximal spanning trees are there?

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5

5. What is the vertex connectivity of a complete bipartite graph $K_{n,n}$ with $n \geq 2$?

(A) n (B) $n - 1$ (C) $2n - 2$ (D) $2n$ (E) $n + 1$

6. How many edges does the smallest Eulerian graph with 7 vertices have?

(A) 0   (B) 6   (C) 7   (D) 8   (E) 10

7. What is the minimum number of vertices required for an Eulerian graph with 13 edges?

    (A) 3   (B) 4   (C) 5   (D) 6   (E) 7

8.   • (A) How many Hamiltonian paths are there in $K_{4,5}$?

     (A) 0   (B) 720   (C) 1440   (D) 4!5!   (E) 9!

   • (B) How many Hamiltonian paths are there in K_5?

     (A) 0   (B) 12   (C) 48   (D) 60   (E) 120

9. Determine the number of all connected regular planar graphs G such that the number of regions in a planar embedding of G equals its order.

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

10. How many 5-regular maximal planar graphs are there?[1]

(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

---

[1]A planar graph is *maximal* if no more edges can be added to it without losing its planarity.

Problem-Solving Question (5%)

Question: We have a labeled graph K_n, where $n > 1$ and the vertex set $[n]$. Find the number of spanning trees that contain the edge 12.