King Fahd University of Petroleum and Minerals

Mathematics Department

Math 471 Exam I, 1st Semester (221),

Net Time Allowed: 90 minutes

October 3rd , 2022

Name:

ID No.:

Section NO.:

Please:

1. Write clearly with a **pen or dark pencil** in the **designed area for each question.**
2. **Fill your info clearly**, and write your **ID NO** in each page in the right corner**.**
3. Show **all** your steps, no credit will be given to wrong steps.

1) Let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$, where $a \neq 0$. Find the **factorization $\boldsymbol{LDL^t}$** for A.

2) If the following sequences of operations were performed on $A,\ n \times n$ matrix:
$E_2 - 3E_1 \rightarrow E_2$, $E_2 \leftrightarrow E_3$, $2E_1 \rightarrow E_1$, $-2E_2 \rightarrow E_2$, $E_3 - \frac{1}{2}E_1 \rightarrow E_3$, and $\frac{1}{3}E_3 \rightarrow E_3$.

What is determent of the new matrix in term of $|A|$?

3) Using **Gaussian elimination with scaled partial pivoting** and three-digit rounding arithmetic to solve the linear system:

$$3.03x_1 - 12.1x_2 + 14x_3 = -119$$
$$-3.03x_1 + 12.1x_2 - 7x_3 = 120$$
$$6.11x_1 - 14.2x_2 + 21x_3 = -139$$

4) Show that the equation for a straight line through two points (x_1, y_1) and (x_2, y_2) in the plan can be written as $\begin{vmatrix} 1 & x & y \\ 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \end{vmatrix} = 0$.

5) Compute the $P^t\boldsymbol{LU}$ **factorization** of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 5 & 2 \\ 2 & -2 & 0 \end{bmatrix}$.

6) Show that $A = \begin{pmatrix} 0 & a & 0 & 0 & 0 \\ b & 0 & c & 0 & 0 \\ 0 & d & 0 & e & 0 \\ 0 & 0 & f & 0 & g \\ 0 & 0 & 0 & h & 0 \end{pmatrix}$ is not invertible for any value of the entries

7) Show if A^{-1} exist, then it is unique.

8) How many multiplications and additions are required to determine a sum of the form

$$\sum_{i=1}^{n} \sum_{j=1}^{i} x_i y_j$$

9) Let $A = \begin{bmatrix} \alpha & 1 & 0 \\ \beta & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$. Find all values of α and β for which

a. A is **singular**.
b. A is **strictly diagonally dominant**.
c. A is **positive definite**.