King Fahd University of Petroleum and Minerals

Mathematics Department

Math 471 Exam II, 1st Semester (221),

Net Time Allowed: 90 minutes

November 10, 2022

Name:

ID No.:

Section NO.:

Please:

1. Write clearly with a **pen or dark pencil** in the **designed area for each question.**
2. **Fill your info clearly**, and write your **ID NO** in each page in the right corner**.**
3. Show **all** your steps, no credit will be given to wrong steps.

1) Let A be $n \times n$ matrix and $x \in \mathbb{R}^n$. Show that

$$\|Ax\|_\infty \leq n^{\frac{1}{2}} \|A\|_2 \|x\|_\infty$$

2) Find the **linear least squares polynomial** approximation for $f(x) = x^3$ on the interval $[-1, 1]$.

3) Let $A = \begin{bmatrix} 3 & 2 \\ 3 & -2 \end{bmatrix}$.

a. Find the **eigenvalues** and **eigenvectors**?

b. Find the **condition number** for A with relative to norm two?

4) Given the linear system

$$2x + y = 3$$

$$x + 3y = 2.$$

a) Find the first two iterations of Jacobi, using $\boldsymbol{x}^0 = \mathbf{0}$,

b) Find the first two iterations of SOR method with $\omega = 1.05$ and using $\boldsymbol{x}^0 = \mathbf{0}$,

c) Find the optimal ω for SOR for that linear system.

5) Let $\left\{1, x, x^2 - \frac{1}{3}, x^3 - \frac{3}{5}x\right\}$ is a set of orthogonal polynomials on [-1,1] with respect to the weight function $w = 1$. Use that set to find a **least squares polynomial** approximation **of degree two** for $f(x) = sin\, x$ with $w = 1$ on $[-1, 1]$. Given that, $\int_0^1 \sin x\, dx = 1 - \cos(1)$, $\int_0^1 x \sin x\, dx = \sin(1) - \cos(1)$, and $\int_0^1 x^2 \sin x\, dx = 2\sin(1) + \cos(1) - 2$.

6) Given that $A = \begin{bmatrix} 1.01 & 0.99 \\ 0.099 & 1.01 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$. If $\|A^{-1}\|_\infty$ and an approximation $\tilde{x} = (2, 0)^t$ is used.

   a) Find a bound for the relative error.
   b) Using the iterative refinement method, find the 2^{nd} iteration for the approximation.

7) Consider the collection of data

x	1.00	1.25	1.50	1.75	2.00
y	5.10	5.79	6.53	7.45	8.46

Construct the least squares approximation of the form $y = be^{ax}$. (Show all necessary work, don't use the built-in command in the calculator)

8) Let λ be an eigenvalue of $n \times n$ matrix A and $\boldsymbol{x} \neq 0$ be an associated eigenvector. Let $\alpha \neq \lambda$ be given. Show that if $A - \alpha\, I$ is nonsingular, then $\frac{1}{\lambda - \alpha}$ is an eigenvalue of $(A - \alpha\, I)^{-1}$ with eigenvector $\boldsymbol{x}$.

*Please, you need to write in the bottom of the question where you need more space “**Go to page 10**”.

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*Please, you need to write in the bottom of the question where you need more space “**Go to page 12**”.