King Fahd University of Petroleum and Minerals

Mathematics Department

Math 471 Final Exam, 1st Semester (221),

Net Time Allowed: 180 minutes

25 Dec. 20022

Name:

ID No.:

Please:

1. Write clearly with a **pen or dark pencil** in the **designed area for each question.**
2. **Fill your info clearly**, and write your **ID NO** in each page in the right corner**.**
3. Show **all** your steps, no credit will be given to wrong steps.

1) Determine the **Geršgorin circles** for the matrix

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 1 & 3 & 2 \end{bmatrix}$$

10 points

and use these to find bounds for the **spectral radius** of A.

2) Consider $\mathbf{v}_1 = (1,1,0)^t,\ \mathbf{v}_2 = (1,0,1)^t,\ \mathbf{v}_3 = (0,1,1)^t$

10 points

a) Show that the set is linearly independent
b) Use the **Gram-Schmidt process** to find a set of orthogonal vectors.

3) Let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and use $x^{(0)} = (1,0)^t$.

10 points

a. Find the first <u>two</u> iterations obtained by the **Power method** applied to A
b. Find the first <u>two</u> iterations obtained by the **Symmetric Power method** applied to A

4) Apply **Householder transformations** to

$$\begin{bmatrix} 12 & 10 & 4 \\ 10 & 8 & -5 \\ 4 & -5 & 3 \end{bmatrix}$$

to place it in tridiagonal form.

10 points

5) Apply <u>two</u> iterations of the **QR method** to this matrix

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

10 points

6) Given the data

10 points

x_i	1.0	2.0	3.0	4.0	5.0
y_i	1.3	3.5	4.2	5.0	7.0

,

Use the **singular value decomposition** technique to determine the **least squares polynomial** of degree 1 where

$$s = \begin{bmatrix} 7.69 & 0.00 \\ 0.00 & 0.92 \\ 0.00 & 0.00 \\ 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix}, \quad V = \begin{bmatrix} 0.27 & 0.96 \\ 0.96 & -0.27 \end{bmatrix}, \quad U = \begin{bmatrix} 0.16 & 0.76 & -0.41 & -0.36 & -0.31 \\ 0.29 & 0.47 & 0.07 & 0.39 & 0.73 \\ 0.41 & 0.18 & 0.84 & -0.20 & -0.24 \\ 0.54 & -0.11 & -0.22 & 0.65 & -0.47 \\ 0.66 & -0.40 & -0.27 & -0.49 & 0.29 \end{bmatrix}.$$

7) Determine the **rank** and the **singular values** of

10 points

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 0 \\ -1 & 1 \end{bmatrix}$$

8) Consider $A = \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$

a. Show that A is **positive definite**.

b. Use the **Cholesky** Algorithm to find a factorization of the form LL^t for A.

10 points

9) Consider the linear system

$$4x - y - z = 1,$$

$$-x + 4y - w = 2,$$

$$-x + 4z - w = 0,$$

$$-y - z + 4w = 1$$

Find an approximation to the solution by using **the SOR Method** with the optimal ω=1.07. Perform only two iterations. Use $x^{(0)} = (0,0,0,0)^t$.

10 points

10) Use **Gaussian elimination scaled partial** pivoting and three-digit arithmetic chopping to solve the following linear systems

10 points

$$58.9x_1 + 0.03x_2 = 59.2,$$
$$-6.10x_1 + 5.31x_2 = 47.0.$$

11) **True** or **False**: If true, explain why. If false, give a counterexample.

20 points

a. If A is a nonsingular symmetric matrix, then A^{-1} is also symmetric. **( )**

b. The product of two tridiagonal matrices is tridiagonal. **( )**

c. If A and B are two square matrices, then $|A + B| = |A| + |B|$. **( )**

d. If A and B are similar matrices, then $\|A\|_\infty = \|B\|_\infty$. **( )**

e. Applying an elementary row operation to an orthogonal matrix produces an orthogonal matrix. **( )**

f. If λ is an eigenvalue of both A and B, then it is an eigenvalue of the sum A + B. **(   )**

g. A positive definite matrix is strictly diagonally dominant. **(   )**

h. Every permutation matrix satisfies $P^2 = \mathbf{I}$. **(   )**

i. The condition number of the identity relative at any norm matrix is 0. **(   )**

j. The singular values of A^t are the same as the singular values of A. **(   )**