King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 472 Major Exam 1 (211)
Time Allowed : 120 Minutes

Name: ______________________________ ID#: ____________________

Instructor: ______________________ Sec #: __________ Serial #: __________

- Mobiles are not allowed in this exam.
- Answers should be neat, clear, and legible.
- Show all steps
- Write your answers in six significant digits

Question #	Marks	Maximum Marks
1		14
2		14
3		10
4		15
5		12
6		10
Total		75

Q1 (4+5+5 points) Consider $f(x) = e^x$ and the nodes $x_0 = 0$, $x_1 = 1$, $x_2 = 2$ and $x_3 = 3$. Use Forward divided differences to construct the polynomials of degree ONE, TWO, and THREE.

Note: $\Delta f(x_0) = f(x_1) - f(x_0)$, and $P_n(x) = f[x_0] + \sum_{k=1}^{n} \begin{pmatrix} s \\ k \end{pmatrix} \Delta^k f(x_0)$.

Table 1: Time, Distance and Speed data of a moving car

Time	0	2
Distance	0	100
Speed	10	50

Q2 (10+2+2 points) Data given in table 1 is obtained from a moving car on a straight road.

(a) Use divided differences to construct a Hermite polynomial for the data.

(b) Use that polynomial to predict the distance and speed of the car at $t = 1$ seconds.

(c) Find the predicted maximum speed for the car during the time period $[0, 2]$.

Q3 (10 points) Use Lagrange interpolating polynomial to derive the Trapezoidal rule

$$\int_a^b f(x)dx = \frac{h}{2}[f(a) + f(b)] - \frac{h^3}{12}f''(\xi)$$

Q4 (8+3+4 points) Consider the integral $\int_1^4 x \ln x \, dx$.

(a) Approximate the integral using composite Simpson's rule with N=6.

(b) Compute the absolute error between Exact and Numerical solutions.

(c) Determine value of h that will assure value of error less than 10^{-5}. Use $Error = -\frac{b-a}{180} h^4 f^4(\xi)$

Q5 (6+6 points) Use the following formula and Richardson's extrapolation to construct formulas of order $O(h^4)$ and $O(h^6)$

$$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h} - \frac{h^2}{3!}f^{(3)}(x_0) - \frac{h^4}{5!}f^{(5)}(x_0) - \frac{h^6}{7!}f^{(7)}(x_0) - \cdots.$$

Use these formula to approximate $f'(2)$ when $f(x) = xe^x$ and $h = 0.2$. Find absolute error for these formulas. Use 6 decimal places in your calculations.

Q6 (10 points) Use Gaussian quadrature of order TWO to approximate the integral $\int_{1}^{1.5} x^2 e^{-x}\, dx$.

Note: $t_i = \pm\frac{\sqrt{3}}{3}$ and $x = \frac{(b-a)t + a + b}{2}$