King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 472 Major Exam 2 (211)
Time Allowed : 120 Minutes

Name: ______________________ ID#: ______________

Instructor: ______________________ Sec #: ________ Serial #: ________

- Mobiles are not allowed in this exam.
- Answers should be neat, clear, and legible.
- Show all steps
- Write your answers in six significant digits

Question #	Marks	Maximum Marks
1		22
2		12
3		13
4		14
5		14
Total		75

Q1 (5+6+6+5 points) Consider the initial value problem

$$y' = t^{-2}(\sin 2t - 2ty), \quad 1 \le t \le 2, \quad y(1) = 2$$

(a) Show that the IVP has a unique solution

(b) Find its exact solution

(c) Consider a perturbed problem and find its exact solution

(d) Show that the IVP is well posed

Note: Write perturbed IVP with δ and δ_0 such that $\delta < \epsilon$ and $\delta_0 < \epsilon$ for some $\epsilon > 0$.

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Q2 (6+6 points) (a) Derive Euler's method using Taylor's series to approximate the solution of an IVP at t_{i+1}.

(b) Compute the error bound for the IVP

$$y' = te^{3t} - 2y, \quad 0 \le t \le 1, \quad y(0) = 0, \quad h = 0.1$$

with the exact solution $y(t) = \frac{1}{5}te^{3t} - \frac{1}{25}e^{3t} + \frac{1}{25}e^{-2t}$.

Q3 (13 points) Setup a Taylor's method of order 4 for the following IVP

$$y' = e^{2t} - y, \quad 0 \le t \le 1, \quad y(0) = 1$$

Write your answer as: $w_{i+1} = w_i + F(h)e^{2t_i} + G(h)w_i$

Note: $T^{(n)}(t, y) = f(t, y) + \dfrac{h}{2!}f'(t, y) + \dfrac{h^2}{3!}f''(t, y) + \cdots + \dfrac{h^{n-1}}{n!}f^{(n-1)}(t, y)$

Q4 (6+8 points) (a) Derive a 3-Step Adams-Bashforth formula using backward differences.

(b) Use it to approximate $y(2.3)$ using exact starting values for the IVP

$$y' = -y + t\sqrt{y}, \quad 2 \leq t \leq 3, \quad y(2) = 2, \quad h = 0.1$$

The exact solution is $y(t) = (t - 2 + \sqrt{2}e^{1-t/2})^2$

Q5 (7+7 points) (a) For the following IVP

$$y'' - 3y' + 2y = 6e^{-t}, \quad 0 \leq t \leq 1, \quad y(0) = 2, y'(0) = 3$$

write a system of first order initial value problems.

(b) Approximate $y(0.25)$ and $y'(0.25)$ with $h = 0.25$ using Euler's method.