King Fahd University of Petroleum and Minerals
Department of Mathematics

MATH 472

Exam-II (Dr. Faisal A. Fairag)
March 25, 2023

NAME:		**ID:**	

Q		Points
1		13
2		9
3		8
4		7
5		10
6		7
7		6
8		10
9		8
10		6
11		6
12		10
TOT		100

PART-1

1) Given the data:

x_i	4.0	4.2	4.5	4.7	5.1	5.5	5.9	6.3	6.8	7.1
y_i	102.56	113.18	130.11	142.05	167.53	195.14	224.87	256.73	299.50	326.72

```
x = [4.0 4.2 4.5 4.7 5.1 5.5 5.9 6.3 6.8 7.1];
y = [102.56 113.18 130.11 142.05 167.53 195.14 224.87 256.73 299.50 326.72];
```

A) Construct the least squares polynomial $\boldsymbol{p_3(x)}$ of degree 3

$\boldsymbol{p_3(x)} =$

B) compute the error Er_3 in part (a)

$Er_3 =$

C) Construct the least squares approximation of the form $\boldsymbol{q(x) = \frac{1}{\alpha}\left(\frac{\beta + x}{x}\right)}$

$\alpha =$	$\beta =$

D) compute the error Er_q in part (C)

$Er_q =$

E) Which model best fits the data?

F)

$p_3(x)$	OR	$q(x)$

G) Graph the data points, the function $q(x)$ obtained in part (d), and the polynomial $p_3(x)$ obtained in part (a) in one plot. Upload your live editor file as .pdf and. mlx via Blackboard. This file should display coefficients of $p_3(x), Er_3, \alpha, \beta, Er_q$ and all the graphs. **(IN ONE FILE)**

2) a) The following list contains homework grades and the final-examination grades for 30 numerical analysis students.

Homework	Final Exam		Homework	Final Exam
302	45		323	83
325	72		337	99
285	54		337	70
339	54		304	62
334	79		319	66
322	65		234	51
331	99		337	53
279	63		351	100
316	65		339	67
347	99		343	83
343	83		314	42
290	74		344	79
326	76		185	59
233	57		340	75
254	45		316	45

```
Homework=[302 325 285 339 334 322 331 279 316 347 343 290 326 233 254 323 337 337 ...
    304 319 234 337 351 339 343 314 344 185 340 316];
FinalExam=[45 72 54 54 79 65 99 63 65 99 83 74 76 57 45 83 99 70 62 66 51 53 100 ...
    67 83 42 79 59 74 45];
```

A) Find the equation of the least squares line for these data.
(Let tot = Homework + Final Exam) $\boldsymbol{tot = \alpha * Homework + \beta}$

α=		$\beta =$	

B) compute the correlation coefficient.

Correlation Coefficient:	

C) Plot the data points and the least square line then upload your live editor file as .mlx format and .pdf format. The file should display least square coefficients, correlation coefficient and the plot. **(IN ONE FILE)**

3) A) Use the classical Legendre polynomials to compute the least square polynomial approximation $q_4(x)$ of degree 4 to the function $\boldsymbol{f(x) = \frac{\ln(x+3)}{x+1.2}}$

$$\boldsymbol{q_4(x) = C_0\, P_0(x) + C_1\, P_1(x) + C_2\, P_2(x) + C_3\, P_3(x) + C_4\, P_4(x)}$$

$C_0 =$		$C_3 =$	
$C_1 =$		$C_4 =$	
$C_2 =$			

B) Graph the functions $f(x)$ and $q_4(x)$ in one plot then upload your live editor file as pdf format and. mlx via Blackboard. This file should display all coefficients of the polynomial $q_4(x)$. **(IN ONE FILE)**

4) A) Use the Gram-Schmidt procedure to calculate $H_0(x), H_1(x), H_2(x)$ and $H_3(x)$ where $\{H_0(x), H_1(x), H_2(x), H_3(x)\}$ is an orthogonal set of polynomials on the interval $(-\infty, \infty)$ with respect to the weight function $\boldsymbol{w(x) = e^{-x^2}}$ and the leading coefficient of $H_j(x)$ is equal to 1. The polynomials obtained from this procedure are called the **Hermite polynomials.**

$H_0(x) =$	
$H_1(x) =$	
$H_2(x) =$	
$H_3(x) =$	

B) Upload your live editor file as .pdf format and .mlx format via Blackboard
(IN ONE FILE)

5) A) Use the Fast Fourier Transform Algorithm (fft) to determine the discrete least squares trigonometric polynomial $S_3(x)$ on the interval $[-\pi, \pi]$ for the data

x	$-\pi$	$-7\pi/9$	$-5\pi/9$	$-3\pi/9$	$-\pi/9$	$\pi/9$	$3\pi/9$	$5\pi/9$	$7\pi/9$
y	-4.000	0.3251	1.1679	-1.8660	4.0374	4.7214	-0.1340	3.1375	1.6107

```
y = [-4.0000 0.3251 1.1679 -1.8660 4.0374 4.7214 -0.1340 3.1375 1.6107];
```

$S_3(x) =$

B) Graph the function $S_3(x)$ and the data points then upload your live editor file as .pdf and .mlx via Blackboard. This file should display all the coefficients of the function $S_3(x)$ **(IN ONE FILE)**

6) A) Construct a parametric cubic spline $s(x, y)$ with natural end points using the following data points then estimate the value of y when $x = 300$.

x = [902 920 917 798 764 752 758 776 799 779 752 619 535 509 514 557 605 612 508 430 424 436 424 346 302 222 199 201 239 250 210 64 10 54 123 112 80 88 106 96];

y =[304 253 208 126 146 173 211 237 224 190 173 115 109 123 179 219 224 199 141 124 138 175 140 95 103 263 443 561 599 521 401 341 402 518 498 456 466 486 480 469];

$y(x = 300) =$

B) Plot the curve $S(x, y)$ and the points (x_i, y_i) then upload your live editor file as .pdf format and .mlx format. This file should display the value of $y(300)$. **(IN ONE FILE)**

PART-2

7) A natural cubic spline S is defined by

$$S(x) = \begin{cases} S_0(x) = 2 + B(x-1) + D(x-1)^3, & if\ 1 \le x < 2 \\ S_1(x) = 2 + b(x-2) + \frac{3}{2}(x-2)^2 + d(x-2)^3, & if\ 2 \le x \le 3 \end{cases}$$

If S interpolates the data $(1,2), (2,2),$ and $(3,t)$, find B, D, b, d and t.

8) If $w = \text{fft}(v)$ where $v = [y, 1, 1, y]^T$ *and* $w = [c + 2, d - 4, c - 2, q - 3]^T$ then find c, y, d, q
(where fft is the built-in MATLAB command)

9) Find the continuous least squares polynomial $S_2(x)$ for $f(x) = x + x^2$ on $[-\pi, \pi]$
(Show all your work)

10) Show that if $\{\phi_0(x), \phi_1(x), \cdots, \phi_n(x)\}$ is an orthogonal set of functions on $[a, b]$ with respect to the weight function $w(x)$, then $\{\phi_0(x), \phi_1(x), \cdots, \phi_n(x)\}$ is a linearly independent set

11) Let $v_1(x) = x\,P_5(x), v_2(x) = x\,P_8(x)$ and $v_3(x) = x\,P_{11}(x)$ where $\{P_j(x)\}_{j=0}^{12}$ are the classical Legendre polynomials. Is the set of polynomials $\{v_1(x), v_2(x), v_3(x)\}$ Orthogonal? If your answer is YES, then show that $\{v_1(x), v_2(x), v_3(x)\}$ is an orthogonal set. If your answer is NO, then write $v_3(x)$ as a linear combination of $v_1(x)$ and $v_2(x)$. (i.e) $v_3(x) = c_1\,v_1(x) + c_2\,v_2(x)$

12) Let $C = [1, 2, 3, 4, 5, 6, 7, 8]^T$ and the vector y =ifft(c)

If the fourth element in the vector y satisfy the following

$y_4 = (F_4 * \boldsymbol{C_e})_4 + \boldsymbol{k}\,(F_4 * \boldsymbol{C_o})_{\boldsymbol{4}}$ where F_4 is the Fourie Matrix of size 4×4.

Then find the two vectors $\boldsymbol{C_e}$ and $\boldsymbol{C_o}$ and the constant $\boldsymbol{k}$

$\boldsymbol{C_e} =$	$\boldsymbol{C_o} =$	**Hint:** The operation-reduction feature of the fast Fourier results from dividing the vector into two smaller vectors and the matrix-vector product of size 2^n reduces to two matrix-vector products of size 2^{n-1}. This process continues to subdivide the problem until we get a vector of length 1.

$\boldsymbol{k}$	