

**Name/ID:**

**Question 1:** Consider the linear system:

$$x + y + z = 0$$

$$x + 2y + 2z = 0$$

$$3x + by + 4z = a$$

Use Gaussian Elimination to find all possible values of  $a$  and  $b$  that make the system to

- a) have a unique solution,
- b) have infinitely many solutions,
- c) be inconsistent [no solution].

**Question 2:** Write an  $LU$  factorization without pivoting for

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 2 & 3 \end{pmatrix}.$$

**Question 3:** Given that  $M$  is a “large” number, use an appropriate method for obtaining an accurate numerical solution for the system:

$$x + My = M$$

$$x + y = 2.$$

**Question 4** A function  $f(x) = x^3 + x^2 + ax + b$  satisfies the points  $P_1(1, 3)$ ,  $P_2(-1, -3)$ .

- a) Write a linear system for finding the coefficients  $a$  and  $b$ .
- b) Use Gaussian Elimination to solve the linear system.

**Question 5:** Solve the linear system using Front/Back Substitutions,

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 3 \end{pmatrix}.$$

**Question 6:** Consider the following matrix:

$$A = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Use the properties of Block Matrices to find

a)  $\det(A)$

b)  $A^{-1}$

c)  $(x_3, x_4, x_5)$  satisfying  $AX = (1 \ 2 \ 0 \ -1 \ -2)^t$