😊**<u>Exam II -Math 503 -241-KFUPM-Dr. Jamal</u>**😊

<u>Name/ID:</u>

<u>Question 1</u>: Consider the following matrix:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

a) Use Echelon Form to find a basis for $Row(A)$, and a basis for $Col(A)$.
b) Find an eigenvector of A corresponding to its eigenvalue $\lambda = 2$.

<u>Question 2</u>: Use eigenpairs to find a decomposition VDV^{-1} for the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}.$$

Question 3: Let A be a matrix whose eigenpairs are given by

$$\lambda_1 = 1, \qquad V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad \lambda_2 = 2, \qquad V_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Find: a) the matrix A, b) the matrix e^A

Question 4: An eigenpair of a nonsingular matrix A is $\lambda = \pi$, $V = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

Find an eigenpair for: a) A^{-1} b) $\sin A$

Question 5: Use the Least-Squares Method to find the best-fit function of the form

$$f(x) = a + bx^2 + x^3$$

for the points $P(0, 2)$, $\mathrm{R}(-1, 0)$, and $Q(1, 4)$.

Question 6: Let L be a lower triangular matrix of order N, and whose diagonal elements are ones. Find the total number of arithmetic operations for finding for solving the system $LX = b$.

Question 7: Enjoy the following interesting little questions:

a) Let $V \in R^3$ a column vector, where $V \neq O$. Find $Rank(VV^T)$.

b) Let $W \in R^n$. Find the number of arithmetic operations to compute $(W^TW)^{100}$.

c) A matrix $A_{60\times20}$ has exactly 10 linearly independent columns.

- Then, A has exactly how many linearly independent rows?

- Is A^TA invertible in this case?

d) Let B be a 1000×40 matrix. Then, the matrix B^TB is invertible (nonsingular) if $Rank(A) =$?

e) If $A = A^{-1}$, then what can be concluded about A?

Enjoy it 😊