MATH503-TERM241-KFUPM

NAME/ID: ***Enjoy it ☺***

Question 1: Solve the linear system using Forward/Back Substitutions,

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 3 \end{pmatrix}.$$

Question 2: Let A be an invertible matrix,

$$A = \begin{pmatrix} a & b & 0 & 1 & 0 \\ c & d & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Use the properties of Block Matrices to find

a) $\det(A)$
b) (x_3, x_4, x_5) satisfying $AX = (-1 \quad -2 \quad 0 \quad 1 \quad 2)^T$

Question 3: Given that M is a "Large" number, use a pivoting technique for obtaining an accurate numerical solution for the system:

$$x + y = 11, \qquad x + My = M.$$

Question 4: Consider

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

a) Write the economical singular value decomposition (SVD) of the matrix
b) Find the condition number of A.

Question 5: Consider the linear system $AX = b$, where

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 4 \\ -2 & 10 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}.$$

a) Find the QR-Factorization of A.
b) Write the method of the *QR*-factorization for solving the linear system.

Question 6: Consider the matrix

$$A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}.$$

a) Diagonalize *A* in the form $A = VDV^{-1}$.
b) Use the diagonalization to find A^N

Question 7: Answer the following questions:

a) Let $W \in R^N$. Find the number of arithmetic operations to compute $(W^T W)W$.

b) A matrix $A_{30\times 40}$ has exactly 30 linearly independent rows.
   - Then, A has exactly how many linearly independent columns?
   - Is $A^T A$ invertible in this case? Justify your answer.

c) If $B = B^{-1}$, find all possible values of $\det(B)$.

Question 8: Consider the linear system:

$$AX = b,$$

where $A \in R^{m \times n}$, $m > n$, with $Rank(A) = n$, and $b \in R^{m}$.

a) Write the solution of the system using the SVD-factorization.
b) Write the solution of the system using the Normal Equation.
c) Show that the two solutions are the same.

Question 9: Let $A = QR$ be a QR-factorization, where A is nonsingular.

a) Use the QR-factorization of A to find A^{-1}.
b) Use the QR-factorization of A to find $\det(A)$.

Question 10: Let A be an invertible matrix, and O be the zero matrix of order N,

$$B = \begin{bmatrix} A & A & O \\ A & A & A \\ O & A & A \end{bmatrix}.$$

Use Gaussian Eliminations to find B^{-1}.