Mathematics for Data Science(MATH-503)
Mid Term Examination

Instructions:

1. Write your name in CAPITAL letters.
2. Attempt all questions.
3. Each Multiple-Choice question has 5 points.

Name:– ID: – – – – – – – – – – – –

1. (19 points) Given the matrix:

$$A = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 1 & -1 & 0 & 0 \\ 1 & 5 & 2 & 6 \end{bmatrix}$$

   (a) Find row echelon form for A.

(b) Identify the columns that correspond to redundant data attributes.

(c) Determine the dimension of the vector space spanned by the columns of A.

(d) How many columns are necessary for fitting $A\mathbf{x} = \mathbf{b}$?

(e) How many vectors are needed to span the kernel of A?

(f) Write down the basis for $\mathrm{Im}(A)$.

2. (16 points) Consider the vectors:

$$\mathbf{x}_1 = (1, -1, -2, 2)^T, \quad \mathbf{x}_2 = (2, 1, 2, -1)^T$$

(a) Compute the ℓ_1, ℓ_2, and ℓ_∞ distances between $\mathbf{x}_1$ and $\mathbf{x}_2$.

(b) Compute the cosine of the angle between $\mathbf{x}_1$ and $\mathbf{x}_2$.

3. (9 points) For each set below, determine whether it is orthogonal, orthonormal, or neither. Show your steps.

(a) $\mathcal{A}_1 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \right\}$

(b) $\mathcal{A}_2 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ -1 \end{bmatrix} \right\}$

(c) $\mathcal{A}_3 = \left\{ \frac{1}{2}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\-1\\0\end{bmatrix} \right\}$

4. (16 points) Consider fitting the line $d = m_0 + m_1 x$ to the data:

x	-1	0	1	0
d	2	-1	4	1

(a) Formulate the linear system to be solved for m_0, m_1.

(b) Write the linear system in matrix form.

(c) Use the normal equation to write down the linear system.

(d) Solve the normal system.