King Fahd University of Petroleum and Minerals
College of Computing and Mathematics
Department of Mathematics
MATH 513 - Midterm Exam

AY 2024-2025 (Term 241)

Name: .... **ID number:** ....

- Textbook, notes, mobiles and smart devices are not allowed in this exam.

- Write neatly and legibly. You may lose points for messy work.

- Show all your work. No points for answers without justification.

Question	Marks	Max Marks
1		15
2		20
3		10
4		20
5		15
Total		80

Question 1

Find the eigenvalues and corresponding eigenvectors for the following matrix:

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix},$$

and determine whether the matrix is diagonalizable or not.

Question 2

Given the following function:

$$f(t) = \begin{cases} 0, & 0 < t < \frac{\pi}{2} \\ 2, & \frac{\pi}{2} < t < \pi \end{cases}$$

(a) Extend $f(t)$ as an even function.

(b) Find the Fourier cosine series of $f(t)$.

Question 3

Given that the Fourier series for the odd function:

$$f(t) = \begin{cases} 2t + t^2, & -2 \leq t \leq 0, \\ 2t - t^2, & 0 \leq t \leq 2, \end{cases}$$

is

$$f(t) = \frac{32}{\pi^3} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \sin\left(\frac{(2n-1)\pi t}{2}\right),$$

find the sum of the series

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^6}$$

Question 4

Find the eigenvalues and eigenfunctions for the following regular Sturm-Liouville problem:

$$y'' + \lambda y = 0, \quad y'(0) = 0, \ y'(\pi) = 0$$

Show that there are only trivial solutions when $\lambda < 0$.

Question 5

Find the first three nonvanishing (nonzero) coefficients in the Legendre polynomial expansion for the function:

$$f(x) = \begin{cases} 0, & -1 < x < 0, \\ x, & 0 < x < 1. \end{cases}$$